

# Learning, Confidence, and Business Cycles\*

COSMIN ILUT  
Duke University & NBER

HIKARU SAIJO  
UC Santa Cruz

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## Abstract

In this paper we argue that a parsimonious propagation mechanism based on information accumulation provides a quantitatively successful business cycle theory that challenges and empirically improves on the conventional view based on an array of real and nominal rigidities. In particular, we build a tractable heterogeneous-firm business cycle model where firms face Knightian uncertainty about their profitability and learn it through production. The cross-sectional mean of firm-level uncertainty is high in recessions because firms invest and hire less. The higher uncertainty reduces agents' confidence and further discourages economic activity. Therefore, the key property of the imperfect information friction is to map fundamental shocks into an as if procyclical equilibrium confidence process. We show how the feedback mechanism endogenously generates co-movement driven by demand shocks, amplified and hump-shaped dynamics, and countercyclical correlated wedges in the equilibrium conditions for labor, risk-free and risky assets. We estimate a rich quantitative model through matching impulse responses of macroeconomic aggregates and asset prices to standard identified shocks. We find that the imperfect information friction improves on conventional models in replicating impulse responses, requires less real and nominal rigidities and predicts magnified responses of economic activity to monetary and fiscal policies.

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\*Email addresses: Ilut [cosmin.ilut@duke.edu](mailto:cosmin.ilut@duke.edu), Saijo [hsaijo@ucsc.edu](mailto:hsaijo@ucsc.edu). We would like to thank George-Marios Angeletos, Yan Bai, Nick Bloom, Fabrice Collard, Stefano Eusepi (our discussant), Jesus Fernandez-Villaverde, Tatsuro Senga, Martin Schneider, Mathieu Taschereau-Dumouchel, Vincenzo Quadrini, Carl Walsh, as well as to seminar and conference participants at the "Ambiguity and its Implications in Finance and Macroeconomics" Workshop, Bank of England, Boston University, Canon Institute for Global Studies, Chicago Fed, Cowles Summer Conference on Macroeconomics, EIEF, Indiana University, Norges Bank, Northwestern, NBER Summer Institute, Richmond Fed, Riksbank, San Francisco Fed, Stanford, UC Santa Cruz, SED Annual Meeting, the SITE conference on Macroeconomics of Uncertainty and Volatility, and Wharton for helpful comments.

# 1 Introduction

Analysts and policy makers generally view aggregate fluctuations as triggered by impulses that vary across historical episodes, such as excessive monetary policy tightening, technological boom-busts, or disturbances in the financial markets. While these impulses differ in their source, business cycles have remarkably consistent patterns, leading to important restrictions on a theory of propagation of shocks. First, there is positive and persistent co-movement of key aggregate quantities, such as hours worked, consumption and investment, which arises robustly from a variety of impulses. Second, this co-movement occurs jointly with predictable cross-equation restrictions between quantities and returns, a pattern that the literature refers to as reduced-form countercyclical labor, consumption and risk premium ‘wedges’.<sup>1</sup>

Conventional quantitative business cycle models typically approach these recurring patterns through the lenses of the New Keynesian (NK) paradigm, where nominal rigidities offer the potential for co-movement out of a broad set of shocks.<sup>2</sup> This view of propagation has been questioned on at least two grounds. First, it relies on estimated nominal rigidities that are typically too large compared to micro data and on a propagation based on sub-optimal monetary policy.<sup>3</sup> Second, even when endowed with a variety of other frictions, quantitative NK models still typically appeal to latent ‘wedges’, as residuals to the optimality conditions for hours, consumption, and capital accumulation. These residuals appear correlated and countercyclical, since the optimality conditions of those models view recessions as periods of ‘unusually’ low hours worked, real interest rates and asset prices.

In this paper we argue that a propagation mechanism based on information accumulation provides a quantitatively successful business cycle theory that challenges and empirically improves on the conventional view. The friction is based on plausible inference difficulties faced by firms, which are uncertain about their own profitability and learn about it through production. The basic reason why the information friction is successful is that it provides a mechanism to map fundamental shocks into procyclical movements in confidence about aggregate conditions. This endogenous correlation propagates a variety of aggregate triggers

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<sup>1</sup>In particular, in a recession, a larger ‘labor wedge’ appears as hours worked are lower than predicted by the comparison of labor productivity to the marginal rate of substitution between consumption and labor, as analyzed through the lenses of standard preferences and technologies (see Shimer (2009) and Chari et al. (2007) for evidence and discussion). At the same time, a higher ‘consumption wedge’ manifests as the risk-free return is unusually low compared to realized future aggregate consumption growth (see Christiano et al. (2005) and Smets and Wouters (2007) as examples for a large literature that uses shocks to the discount factor). Finally, a ‘risk premium wedge’ increases as the excess return on risky assets over the return of risk-free assets is unusually large (see Cochrane (2011) for a review on countercyclical excess returns).

<sup>2</sup>Barro and King (1984) emphasize how in a standard RBC model hours and consumption co-move negatively unless there is a total factor productivity (TFP) or a preference shock to the disutility of working. NK models overturn this impossibility result through countercyclical markups.

<sup>3</sup>See Angeletos (2017) for a critical analysis of the empirical and theoretical underpinnings of NK models.

into fluctuations that have consistently similar patterns: persistent positive co-movement and measured time-varying wedges. Moreover, such a theory is consistent with a view shared by analysts and policymakers that various impulses lead to a similar propagation through which ‘confidence’ or ‘uncertainty’ affect the aggregate economy’s desire to spend, hire and invest.<sup>4</sup>

The endogenous correlation between fundamental shocks and the resulting ‘as if’ confidence process that sustains the equilibrium allocations connects two literatures that suggest the empirical and theoretical appeal of information-driven business cycles. First, the low activity-high uncertainty feedback implied by the friction has been analyzed, in different forms, as a source for business cycle asymmetries, non-linearities, persistence or amplification, in a related learning literature (Caplin and Leahy (1993), van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), Straub and Ulbricht (2016), Fajgelbaum et al. (2017) and Saijo (2017)). While there the feedback typically matters through non-linear dynamics and learning occurs from aggregate market outcomes, we study an endogenous uncertainty mechanism driven by linear dynamics and learning about firm-level profitability. These two properties lead to a tractable characterization and evaluation of the feedback mechanism even within linear, workhorse quantitative models, as well as to novel policy implications. Second, recent work proposes movements in agents’ beliefs, typically modeled as exogenous confidence shocks, as important drivers of business cycles (Angeletos and La’O (2009, 2013), Angeletos et al. (2014), Ilut and Schneider (2014) and Huo and Takayama (2015)). Our analysis provides a theory disciplined by micro and macro moments of the formation of those beliefs, in which the confidence process changes endogenously as a response to the state of the economy.<sup>5</sup>

**Propagation mechanism.** There are three aspects of uncertainty that are key to the proposed mechanism. First, consistent with a view common in the industrial organization literature, a firm is a collection of production lines that have a persistent firm-specific component, as well as temporary independent realizations across lines (see Coad (2007) for a survey). Second, as in models of learning by doing in the firm dynamics literature, similar in spirit to Jovanovic (1982), firms accumulate information about their unobserved profitability through production. Third, perceived uncertainty includes both risk and ambiguity, modeled by the recursive multiple priors preferences axiomatized in Epstein and Schneider (2003b).<sup>6</sup> In particular, we assume that facing a larger estimation uncertainty, the decision-maker is less confident in his probability assessments and entertains a wider set of beliefs about the

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<sup>4</sup>Baker et al. (2016) documents how the word “uncertainty” in leading newspapers and the FOMC’s Beige Book spikes up in recessions. Examples of analysts’ speeches referring to “caution” and “uncertainty” as propagation mechanism in the Great Recession include Blanchard (2009) and Diamond (2010).

<sup>5</sup>See Angeletos and Lian (2016) for a distinct but complementary theory of propagation through endogenous confidence based on a lack of common knowledge, whose main effect is to attenuate general equilibrium effects.

<sup>6</sup>The standard evidence for this extension is the Ellsberg (1961) paradox type of choices. See Bossaerts et al. (2010) and Asparouhova et al. (2015) for recent experimental contributions.

conditional mean of the persistent firm-specific component. The preference representation makes an agent facing lower confidence behave *as if* the true unknown mean becomes worse.<sup>7</sup>

We embed this structure of uncertainty into a standard business cycle model with heterogeneous firms and a representative agent. The structure of uncertainty generates a feedback loop at the firm level: lower production leads to more estimation uncertainty, which in turn shrinks the optimal size of productive inputs. In our model, the firm-level feedback loop aggregates linearly so that recessions are periods of a high cross-sectional mean of firm-level uncertainty because firms *on average* invest and hire less. In turn, the higher uncertainty, and the implied lower confidence, further dampens aggregate activity.<sup>8</sup>

In particular, when confidence is low, the *uncertainty-adjusted* return to working, to consuming and to investing are *jointly* perceived to be low. This leads to a high measured labor wedge since equilibrium hours worked are low even if consumption is low and the realized marginal product of labor is on average unchanged under the econometrician's data generating process. The endogenous countercyclical labor wedge is a crucial property that explains why labor and consumption can both fall following a contractionary supply or demand shock. The low confidence also leads to a high measured consumption wedge because the increased desire to save depresses the real risk-free rate more than the econometrician's measured growth rate of marginal utility. Finally, it makes capital less attractive to hold so investors are compensated in equilibrium by a higher measured excess return. The emergence of these types of financial wedges connects the mechanism to a large literature which takes these forces as exogenous.<sup>9</sup>

**Quantitative analysis.** To quantitatively evaluate how the learning mechanism compares and interacts with other frictions typically used in macroeconomic models, we embed the information friction into a business cycle model with real rigidities (habit formation and investment adjustment costs), nominal rigidities (Calvo-type sticky prices and wages) and financial frictions (costly state verification as in Bernanke et al. (1999)).<sup>10</sup> To discipline the learning parameters we use prior values consistent with David et al. (2015), who estimate a firm-level signal-to-noise ratio relevant for our model, and Ilut and Schneider (2014), who

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<sup>7</sup>This is simply a manifestation of aversion to uncertainty, which lowers the certainty equivalent of the return to production, but, compared to risk, it implies first-order effects of uncertainty on decisions.

<sup>8</sup>Once the equilibrium 'as if' confidence process is taken as given, the mechanisms through which confidence impacts decisions through distortions in all the relevant Euler equations are therefore common to models with exogenous confidence shocks.

<sup>9</sup>The wedges relate the friction to a literature that uses reduced-form 'risk-premium' shocks, starting with Smets and Wouters (2007). See Gust et al. (2017) for a recent contribution emphasizing the quantitative role of these shocks. See Fisher (2015) for an interpretation of these shocks as time-varying preference for liquidity.

<sup>10</sup>We follow the standard approach and include nominal rigidities as the main friction to generate co-movement. Other directions that address the Barro and King (1984) critique include: strategic complementarity in a model with dispersed information (Angeletos and La'O (2013)), heterogeneity in labor supply and consumption across employed and non-employed (Eusepi and Preston (2015)), variable capacity utilization and a large preference complementarity between consumption and hours (Jaimovich and Rebelo (2009)).

bound the size of ambiguity using a model consistency criterion.

We use an estimation procedure that focuses squarely on propagation. Since our friction predicts regular patterns of co-movement and correlated wedges *conditional on any type* of shock, we employ an impulse-response matching estimation. We use standard recursive restrictions in a structural VAR to identify financial, monetary policy and TFP shocks. In addition, we use the observables to construct empirical measures of labor, consumption and excess return wedges.

We first estimate a model featuring *only* the information friction, without additional real or nominal rigidities, by fitting the impulse responses to the financial shock. We do so because this shock is quantitatively important, accounting for a significant fraction of business cycle variation, and informative, as it provides a laboratory for the relevant empirical cross-equation restrictions. We find that this parsimonious model matches the VAR response well. Following a reduction in the credit spread faced by entrepreneurs, the model replicates the persistent and hump-shaped dynamics of aggregate quantities as hours, investment and consumption jointly rise. The model also matches price dynamics: real wages increase, inflation is stable and the real rate increases. Finally, the model is consistent with the observed countercyclical wedges as the labor, consumption and excess return wedges jointly fall.

If we turn off the information friction and re-estimate a rational expectations (RE) model enriched with habit formation, investment adjustment costs, sticky prices and wages, we find that it can match the positive co-movement of real quantities, mostly by appealing to very rigid prices and wages. However, that model predicts consistent deviations from the data: following an expansionary financial shock, inflation is too high, while the real wage and the real interest rate are too low. As a consequence, even if this model has many frictions, it fits the data worse in terms of marginal data density.

Our second estimation experiment is to match impulse responses to all three structural shocks and compare the fit of the standard set of rigidities with a model that *also* includes the information friction. We find that the learning model matches well the three sets of impulse-responses. In contrast, a re-estimated RE model where the information friction is absent fails to replicate key empirical properties. In particular, for the expansionary financial shock, that model predicts flat responses for consumption and the real rates, instead of both rising as in the data. This change in inference compared to the first estimation experiment shows the importance of the additional cross-equation restrictions. We attribute this failing to the RE model requiring a high degree of habit formation to match the negative co-movement between consumption growth and real rate, conditional on a monetary policy shock. The model with learning is instead consistent with some degree of habit needed to match the monetary policy shock, as well as with the positive co-movement of consumption and real rates after the

financial shock. The reason for this joint behavior is the countercyclical consumption wedge.

Second, since the information friction provides the main ingredients for co-movement and wedges, as well as for persistence and hump-shaped dynamics, it significantly reduces the need of additional frictions for fitting the data. In particular, compared to the RE version, the investment adjustment cost is reduced to a third, the average Calvo adjustment period of prices and wages reduces by about half, to 2.7 and 1.8 quarters, respectively. The habit formation parameter is also lower, but not drastically, as the learning model still needs to account for the impact effect of the monetary policy shock.

We use survey data for outside of model validation. First, we analyze the model-implied and empirical impulse responses of dispersion of forecasts about *aggregate* conditions, measured as the range of one quarter ahead forecasts for real GDP growth from the Survey of Professional Forecasters. The model of endogenous confidence replicates well the finding that this dispersion falls when economic activity is stimulated by *any* of the three identified shocks. Second, in our model the lack of confidence about the distribution of firm's profitability is reflected in a set of conditional mean forecasts about the *individual* firm's return on capital. Here we use a series constructed by Senga (2015) on the cross-sectional average dispersion of survey forecasts of firm-level capital return. The model accounts for about 75% of the time-series variation in this micro-level forecast dispersion, and predicts, consistent with the data, that this dispersion and real GDP are negatively correlated.

**Policy implications.** Our model features important policy implications. First, policy changes are transmitted differently compared to an exogenous confidence benchmark. We show that in our estimated model an interest rate rule that responds to the credit spread would significantly lower output variability because it stabilizes the variation in endogenous uncertainty. Indeed, if the confidence process would be counterfactually held fixed at its pre-policy change path, the output variability would be largely unaffected. For fiscal policy we find a significantly larger government spending multiplier also because of its effect on confidence.

Second, there are no information externalities since learning occurs at the firm level. This is in contrast to a case of learning from aggregate market outcomes, where an individual firm does not take into account the externality of generating useful signals for the rest of the economy.<sup>11</sup> Thus, even if policy interventions would affect the aggregate dynamics qualitatively similarly in these two cases, the welfare properties are different. For example, the increased economic activity, and the associated reduction in uncertainty produced by a fiscal stimulus is not welfare increasing in our model.

**Methodological contribution.** Our methodology allows for a tractable aggregation of firm-level uncertainty. In our model, the only difference from the standard setup is that the

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<sup>11</sup>See Caplin and Leahy (1993), Ordoñez (2013) and Fajgelbaum et al. (2017) for a discussion of the information externalities arising in models based on learning from aggregate market outcomes.

representative agent, who owns the portfolio of firms, perceives uncertainty both as risk and ambiguity (Knightian uncertainty) about the distributions of firms' individual productivity. As with risk, the sources of idiosyncratic Knightian uncertainty are independent and identical and the rational representative agent does not evaluate the firms comprising the portfolio in isolation. Indeed, the agent derives wealth through the average dividend from the portfolio of firms, and the continuation utility is a function of wealth.

Since ambiguity is over the conditional means of firm-level profitability, which in equilibrium affects dividends paid out to the representative agent by each firm, uncertainty affects continuation utility by lowering the worst-case mean of firm-level profitability. The agent faces independent and identical sources of uncertainty and therefore acts as if the mean on each source is lower. Therefore, in contrast to the risk case,<sup>12</sup> the *average* dividend obtained on the portfolio of firms, which is the equilibrium object that the representative agent cares about, does not become less uncertain, i.e. characterized by a narrower set of beliefs, as the number of firms increases. In our model, this is simply a manifestation of a general theoretical property of the law of large numbers for i.i.d. ambiguous random variables.<sup>13</sup>

The connection between this decision-theoretical work and macroeconomic modeling has not been yet made in the literature. Our approach therefore opens the door for tractable quantitative models with heterogeneous firms, where firm-specific uncertainty matters even if equilibrium conditions are linearized *both* at the firm and representative household level.<sup>14</sup>

The paper is structured as follows. In Section 2 we introduce our heterogeneous-firm model and discuss the solution method. We describe the potential of endogenous uncertainty as a parsimonious propagation mechanism in Section 3. In Section 4 we add additional rigidities to estimate a model on US aggregate data.

## 2 The model

Our baseline model is a real business cycle model in which, as in the standard framework, firms are owned by a representative household and maximize shareholder value. We augment the standard framework along two key features: the infinitely-lived representative household

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<sup>12</sup>When uncertainty consists only of risk, it lowers that continuation utility by increasing the volatility of consumption. With purely idiosyncratic risk, uncertainty is diversified away since the law of large numbers implies that the variance of consumption tends to zero as the number of firms becomes large.

<sup>13</sup>See Marinacci (1999) or Epstein and Schneider (2003a) for formal treatments.

<sup>14</sup>While some solution methods with heterogeneity are able to use linearization for the aggregate state variables, non-linearities for the firms' policy functions are still generally needed. See Terry (2017) for an analysis of various solution methods. An example of related work with learning about firm-level profitability is Senga (2015), where firms are subject to economy wide shocks to the volatility of their idiosyncratic shocks. There non-linearities in the policy functions arising from decreasing returns to scale produce mis-allocation effects from the evolution of the distribution of firms' production choices and beliefs.

is ambiguity averse and that ambiguity is about the firm-level profitability processes.

## 2.1 Technology

There is a continuum of firms, indexed by  $l \in [0, 1]$ , which act in a monopolistically competitive manner. They rent capital  $K_{l,t-1}$  and hire labor  $H_{l,t}$  to operate  $J_{l,t}$  number of production units, where each unit is indexed by  $j$ . The firm decides how many production units to operate, where  $J_{l,t}$  is given by

$$J_{l,t} = \bar{N} F_{l,t}. \quad (2.1)$$

We define  $F_{l,t} \equiv K_{l,t-1}^\alpha H_{l,t}^{1-\alpha}$  and  $\bar{N}$  is a normalization parameter that controls the level of disaggregation inside a firm. As analyzed below, in our model the uncertainty faced by a firm is invariant to the level of disaggregation.

Each unit  $j$  produces output, which is driven by three components: an economy-wide shock, a firm-specific shock and a unit-specific shock.<sup>15</sup> This output equals

$$x_{l,j,t} = e^{A_t + z_{l,t} + \tilde{v}_{l,j,t}} / \bar{N}, \quad (2.2)$$

where  $A_t$  is an economy-wide technology shock that follows

$$A_t = \rho_A A_{t-1} + \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N(0, \sigma_A^2),$$

$z_{l,t}$  is a firm-specific shock that follows

$$z_{l,t} = \rho_z z_{l,t-1} + \epsilon_{z,l,t}, \quad \epsilon_{z,l,t} \sim N(0, \sigma_z^2), \quad (2.3)$$

and the unit-specific shock follows

$$\tilde{v}_{l,j,t} \sim N(0, \bar{N} \sigma_v^2).$$

The variance of a unit-specific shock is proportionally increasing in  $\bar{N}$ . Intuitively, as each production unit becomes smaller (i.e., as the level of disaggregation increases), the unit-specific component becomes larger compared to the firm-level component.<sup>16</sup>

Since the firm operates  $J_{l,t}$  number of production units given by (2.1) and each unit

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<sup>15</sup>This view of the firm is common in the industrial organization literature (see Coad (2007) for a survey) and has been motivated by observed negative relationship between the size of a firm and its growth rate variance. See Hymer and Pashigian (1962) for an early empirical documentation and Stanley et al. (1996) and Bottazzi and Secchi (2003) for recent studies of this scaling relationship.

<sup>16</sup>The assumption prevents output to be fully-revealing about firm-specific shocks even as we take the limit  $\bar{N} \rightarrow \infty$ . See Fajgelbaum et al. (2017) for a similar approach; in their model, the precision of a signal regarding an aggregate fundamental is decreasing in the number of total firms in the economy.



produces according to (2.2), the firm's total output equals

$$Y_{l,t} = \sum_{j=1}^{J_{l,t}} x_{l,j,t}.$$

Perfectively competitive final-goods firms produce aggregate output  $Y_t$  by combining goods produced by each firm  $l$ :

$$Y_t = \left[ \int_0^1 Y_{l,t}^{\frac{\theta-1}{\theta}} dl \right]^{\frac{\theta}{\theta-1}}, \quad (2.4)$$

where  $\theta$  determines the elasticity of substitution across goods. The demand function for intermediate goods  $l$  is

$$P_{l,t} = \left( \frac{Y_{l,t}}{Y_t} \right)^{-\frac{1}{\theta}},$$

where we normalize the price of final goods  $P_t = 1$ . The revenue for firm  $l$  is then given by

$$P_{l,t} Y_{l,t} = Y_t^{\frac{1}{\theta}} Y_{l,t}^{1-\frac{1}{\theta}}.$$

Because the idiosyncratic shocks  $z_{l,t}$  and  $\tilde{\nu}_{l,j,t}$  can be equivalently interpreted as productivity or demand disturbances by adjusting the relative price  $P_{l,t}$ , we simply refer to  $z_{l,t}$  and  $\tilde{\nu}_{l,j,t}$  as profitability shocks. Note also that the firm-level returns to scale in terms of revenue,  $1 - \frac{1}{\theta}$ , is less than one, which gives us a notion of firm size that is well-defined.

Given production outcomes and its associated costs, firms pay out dividends

$$D_{l,t} = Y_t^{\frac{1}{\theta}} Y_{l,t}^{1-\frac{1}{\theta}} - W_t H_{l,t} - r_t^K K_{l,t-1}, \quad (2.5)$$

where  $W_t$  is the real wage and  $r_t^K$  is the rental rate for capital.

## 2.2 Imperfect information

We assume that agents cannot directly observe the realizations of idiosyncratic shocks  $z_{l,t}$  and  $\tilde{\nu}_{l,j,t}$ . Instead, every agent in the economy observes the economy-wide shocks  $A_t$ , the inputs used for operating production units  $F_{l,t}$ , as well as output  $Y_{l,t}$  and  $x_{l,j,t}$  of each firm  $l$  and production unit  $j$ . The imperfect observability assumption leads to a non-invertibility problem. Agents cannot tell whether an unexpectedly high realization of a production unit's output  $x_{l,j,t}$  is due to the firm being 'better' (an increase in the persistent firm's specific profitability  $z_{l,t}$ ) or just 'lucky' (an increase in the unit-specific shocks  $\tilde{\nu}_{l,j,t}$ ).

Faced with this uncertainty, agents use the available information, including the path of output and inputs, to form estimates on the underlying source of profitability  $z_{l,t}$ . Since the

problem is linear and Gaussian, Bayesian updating using Kalman filter is optimal from the statistical perspective of minimizing the mean square error of the estimates.<sup>17</sup>

The measurement equation of the Kalman filter is given by the following sufficient statistic  $s_{l,t}$  that summarizes observations from all production units within a firm  $l$ :

$$s_{l,t} = z_{l,t} + \nu_{l,t}, \quad (2.6)$$

where the average realization of the unit-specific shock is

$$\nu_{l,t} \equiv \frac{1}{J_{l,t}} \sum_{j=1}^{J_{l,t}} \tilde{\nu}_{l,j,t} \sim N\left(0, \frac{\sigma_\nu^2}{F_{l,t}}\right),$$

and the transition equation for  $z_{l,t}$  is given by (2.3).

The solution to the filtering problem is standard. The one-step-ahead prediction from the period  $t - 1$  estimate  $\tilde{z}_{l,t-1|t-1}$  and its associated error variance  $\Sigma_{l,t-1|t-1}$  are given by

$$\tilde{z}_{l,t|t-1} = \rho_z \tilde{z}_{l,t-1|t-1}; \Sigma_{l,t|t-1} = \rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2.$$

Then, firms update their estimates according to

$$\tilde{z}_{l,t|t} = \tilde{z}_{l,t|t-1} + \frac{\Sigma_{l,t|t-1}}{\Sigma_{l,t|t-1} + F_{l,t}^{-1} \sigma_\nu^2} \cdot (s_{l,t} - \tilde{z}_{l,t|t-1}), \quad (2.7)$$

and the updating rule for variance is

$$\Sigma_{l,t|t} = \left[ \frac{\sigma_\nu^2}{F_{l,t} \Sigma_{l,t|t-1} + \sigma_\nu^2} \right] \Sigma_{l,t|t-1}. \quad (2.8)$$

The dynamics according to the Kalman filter can thus be described as

$$z_{l,t+1} = \rho_z (\tilde{z}_{l,t|t} + u_{l,t}) + \epsilon_{z,l,t+1}, \quad (2.9)$$

where  $u_{l,t}$  is the estimation error of  $z_{l,t}$  and  $u_{l,t} \sim N(0, \Sigma_{l,t|t})$ .

For our purposes, the important feature of the updating formulas is that the variance of the ‘luck’ component, which acts as a noise in the measurement equation (2.6), is decreasing in scale  $F_{l,t}$ . Thus, holding  $\Sigma_{l,t|t-1}$  constant, the posterior estimation uncertainty  $\Sigma_{l,t|t}$  in equation (2.8) increases as the scale decreases. Firm-level output becomes more informative

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<sup>17</sup>In Jovanovic (1982) the firm uses the observed outcome of production to learn about some unobserved technological parameter. In our model, firms learn about their time-varying, persistent profitability. The learning problem of the model with growth is in Appendix 6.5.2, along with other equilibrium conditions.

about the underlying profitability  $z_{l,t}$  as more production units operate.

## 2.3 Household wealth

There is a representative agent whose budget constraint is given by

$$C_t + B_t + I_t + \int P_{l,t}^e \theta_{l,t} dl \leq W_t H_t + r_t^K K_{t-1} + R_{t-1} B_{t-1} + \int (D_{l,t} + P_{l,t}^e) \theta_{l,t-1} dl + T_t,$$

where  $C_t$  is consumption of the final good,  $H_t$  is the amount of labor supplied,  $I_t$  is investment into physical capital,  $B_t$  is the one-period riskless bond,  $R_t$  is the interest rate, and  $T_t$  is a transfer.  $D_{l,t}$  and  $P_{l,t}^e$  are the dividend and price of a unit of share  $\theta_{l,t}$  of firm  $l$ , respectively. Capital stock depreciates at rate  $\delta$  so that it evolves according to

$$K_t = (1 - \delta)K_{t-1} + I_t.$$

The market clearing conditions for labor, bonds and shares are:

$$H_t = \int_0^1 H_{l,t} dl, \quad B_t = 0, \quad \theta_{l,t} = 1.$$

The resource constraint is given by

$$C_t + I_t + G_t = Y_t, \tag{2.10}$$

where  $G_t$  is the government spending and we assume a balanced budget each period ( $G_t = -T_t$ ). For most of the analysis, we assume that government spending is a constant share of output,  $\bar{g} = G_t/Y_t$ .

Notice that our model is one with a typical infinitely-lived representative agent. Therefore, this agent is the relevant decision maker for the firms that operate the technology described in the previous section, since this agent owns in equilibrium the firms, i.e.  $\theta_{l,t} = 1, \forall(t, l)$ . The difference from a standard expected utility model, in which uncertainty is modeled only as risk, is that the decision maker faces ambiguity over the distribution of firm-level productivities, an issue that we take next.

## 2.4 Optimization

We have described so far the firms' production possibilities, the household budget constraint and the available information set. We now present the optimization problems of the representative household and of the firms.

*Imperfect information and ambiguity*

The representative household perceives ambiguity (Knightian uncertainty) about the vector of firm-level productivities  $\{z_{l,t}\}_{l \in [0,1]}$ . We now describe how that ambiguity process evolves. The agent uses observed data to learn about the hidden profitability through the Kalman filter to obtain a benchmark probability distribution. The Kalman filter problem has been described in Section 2.2. Ambiguity is modeled as a one-step ahead *set* of conditional beliefs that consists of alternative probability distributions surrounding the benchmark Kalman filter estimate  $\tilde{z}_{l,t}$  in (2.9) of the form

$$z_{l,t+1} = \rho_z \tilde{z}_{l,t|t} + \mu_{l,t} + \rho_z u_{l,t} + \epsilon_{z,l,t+1}, \quad \mu_{l,t} \in [-a_{l,t}, a_{l,t}] \quad (2.11)$$

In particular, the agent considers a set controlled by a bound on the relative entropy distance. More precisely, the agent only considers the conditional means  $\mu_{l,t}$  that are sufficiently close to the long run average of zero in the sense of relative entropy:

$$\frac{\mu_{l,t}^2}{2\rho_z^2 \Sigma_{l,t|t}} \leq \frac{1}{2} \eta^2, \quad (2.12)$$

where the left hand side is the relative entropy between two normal distributions that share the same variance  $\rho_z^2 \Sigma_{l,t|t}$ , but have different means ( $\mu_{l,t}$  and zero), and  $\eta$  is a parameter that controls the size of the entropy constraint. The entropy constraint (2.12) results in a set  $[-a_{l,t}, a_{l,t}]$  for  $\mu_{l,t}$  in (2.11) that is given by

$$a_{l,t} = \eta \rho_z \sqrt{\Sigma_{l,t|t}}. \quad (2.13)$$

The interpretation of the entropy constraint is that the agent is less confident, i.e. the set of beliefs is larger, when there is more estimation uncertainty. The relative entropy can be thought of as a measure of distance between the two distributions. When uncertainty  $\Sigma_{l,t|t}$  is high, it becomes difficult to distinguish between different processes. As a result, the agent becomes less confident and contemplates wider sets of conditional probabilities.

*Household problem*

We model the household's aversion to ambiguity through recursive multiple priors preferences, which capture an agent's lack of confidence in probability assessments. This lack of confidence is manifested in the set of one step ahead conditional beliefs about each  $z_{l,t+1}$  given in equations (2.11) and (2.13). Collect the exogenous state variables in a vector  $s_t \in S$ . This vector includes the economy-wide shocks  $A_t$ , as well as the cross-sectional distribution of idiosyncratic productivities  $\{z_{l,t}\}_{l \in [0,1]}$ . A household consumption plan  $C$  gives, for every history  $s^t$ , the consumption of the final good  $C_t(s^t)$  and the amount of hours worked  $H_t(s^t)$ .

For a given consumption plan  $C$ , the household recursive multiple priors utility is defined by

$$U_t(C; s^t) = \ln C_t - \frac{H_t^{1+\phi}}{1+\phi} + \beta \min_{\mu_{l,t} \in [-a_{l,t}, a_{l,t}], \forall l} E^\mu[U_{t+1}(C; s^t, s_{t+1})], \quad (2.14)$$

where  $\beta$  is the subjective discount factor and  $\phi$  is the inverse of Frisch labor supply elasticity.<sup>18</sup> We use the expectation operator  $E^\mu[\cdot]$  to make explicit the dependence of expected continuation utility on the conditional means  $\mu_{l,t}$ .

Notice that there is a *cross-sectional distribution of sets* of beliefs over the future  $\{z_{l,t+1}\}_{l \in [0,1]}$ . Indeed, for each firm  $l$ , the agent entertains a set of conditional means  $\mu_{l,t} \in [-a_{l,t}, a_{l,t}]$ . If each set is singleton we obtain the standard expected utility case of separable log utility with those conditional beliefs. When the set is not a singleton, it reflects the assumption that the agent perceives Knightian uncertainty, in addition to the standard risk embedded in the conditional variances about  $z_{l,t+1}$ . As instructed by their preferences, in response to the aversion to that Knightian uncertainty, households take a cautious approach to decision making and act *as if* the true data generating process (DGP) is given by the worst-case conditional belief, which we will denote by  $E_t^*[\cdot]$ .

#### *Uncertainty as risk and ambiguity*

Modeling idiosyncratic uncertainty as both risk and ambiguity matters crucially for its effect on the decision maker's beliefs of continuation utility. Both cases share similar grounds: the sources of uncertainty are independent and identical and the rational decision maker — here the representative agent that owns the firms — does not evaluate the firms comprising the portfolio in isolation. In particular, in both cases, uncertainty over their idiosyncratic profitability matters only if it lowers the agent's continuation utility. That utility is a function of the wealth obtained through the average dividend from the portfolio.

The difference between risk and ambiguity is how it affects continuation utility. With risk only, uncertainty lowers that continuation utility by increasing the volatility of consumption. With purely idiosyncratic risk, uncertainty is diversified away since the law of large numbers implies that the variance of consumption tends to zero as the number of firms becomes large. When uncertainty consists also of ambiguity, it affects utility by making the worst-case probability less favorable to the agent, through its effect on continuation utility in equation (2.14). Since ambiguity is over the conditional means of firm-level profitability, which in equilibrium affects dividends paid out to the agent, uncertainty affects utility by lowering the worst-case mean of firm-level profitability, i.e.  $E_t^* z_{l,t+1}$ . The agent faces independent and identical sources of uncertainty, represented here by the sets of distributions indexed by  $\mu_{l,t}$ ,

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<sup>18</sup>The recursive formulation ensures that preferences are dynamically consistent. Details and axiomatic foundations are in Epstein and Schneider (2003b). Subjective expected utility obtains when the set of beliefs collapses to a singleton.

and therefore acts as if the mean on each source is lower. Therefore, in contrast to the risk case, the average dividend obtained on the portfolio, which is the equilibrium object that the agent cares about, does not become less uncertain, which here means being characterized by a narrower set of beliefs, as the number of firms increases.<sup>19</sup>

Put differently, the assumption of the sources of perceived uncertainty being independent and identical means that the agent is not willing to view a new firm being added to the portfolio as ‘hedging’ out any ambiguity already perceived on that portfolio. Therefore, the agent ends up lacking confidence about the cross-sectional average (i.e. ‘uncertainty over the size of the pie’) as opposed to fully trusting that average but lacking confidence only about its composition (i.e. ‘uncertainty over the shares of the pie’). It is this lack of confidence about the cross-sectional average that makes firm-level uncertainty not disappear through the law of large numbers.

*Worst-case belief and the law of large numbers*

Therefore, once the representative agent correctly understands the effect of firm-level profitability on the continuation utility in equation (2.14), the worst-case belief can be easily solved for at the equilibrium consumption plan. Given the bound in equation (2.13), the worst-case conditional mean for each firm’s  $z_{l,t+1}$  is therefore given by

$$E_t^* z_{l,t+1} = \rho_z \tilde{z}_{l,t|t} - \eta \rho_z \sqrt{\Sigma_{l,t|t}} \quad (2.15)$$

where  $\tilde{z}_{l,t|t}$  is the Kalman filter estimate of the mean obtained in equation (2.7). Thus, the worst-case conditional distribution of each firm’s productivity is

$$z_{l,t+1} \sim N \left( E_t^* z_{l,t+1}, \rho_z^2 \Sigma_{l,t|t} + \sigma_z^2 \right). \quad (2.16)$$

Once the worst-case distribution is determined, it is easy to compute the cross-sectional average realization  $\int z_{l,t+1} dl$ . By the law of large numbers (LLN) this average converges to

$$\int E_t^* z_{l,t+1} dl = -\eta \rho_z \int \sqrt{\Sigma_{l,t|t}} dl. \quad (2.17)$$

where we have used that  $\int \tilde{z}_{l,t|t} dl = 0$ .<sup>20</sup>

Equation (2.17) is a manifestation in this model of the LLN for ambiguous random variables

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<sup>19</sup>See Marinacci (1999) or Epstein and Schneider (2003a) for formal treatments of the law of large numbers for i.i.d. ambiguous random variables. There they show that cross-sectional averages must (almost surely) lie in an interval bounded by the highest and lowest possible cross-sectional mean, and these bounds are tight in the sense that convergence to a narrower interval does not occur. See also Epstein and Schneider (2008) for an application of this argument to pricing a portfolio of firms with ambiguous dividends.

<sup>20</sup>Indeed, since under the true DGP the cross-sectional mean of  $z_{l,t}$  is constant, the cross-sectional mean of the Kalman posterior mean estimate is a constant as well.

analyzed by Marinacci (1999) or Epstein and Schneider (2003a). In particular, now the *average* idiosyncratic uncertainty,  $\int \sqrt{\Sigma_{l,t|t}} dl$ , matters for the *average* worst-case expected  $z_{l,t+1}$ . That formula shows that once ambiguity is taken into account by the agent, the LLN implies that risk itself does not matter anymore for beliefs since the volatility of consumption converges to zero even under the worst-case conditional beliefs.

### *Firms' problem*

Given that in equilibrium the representative agent holds the portfolio of firms, each firm chooses  $H_{l,t}$  and  $K_{l,t-1}$  to maximize shareholder value

$$E_0^* \sum_{t=0}^{\infty} M_0^t D_{l,t}, \quad (2.18)$$

where  $E_0^*$  denotes expectation under the representative agent's worst case probability and  $D_{l,t}$  is given by equation (2.5). The random variables  $M_0^t$  denote state prices of  $t$ -period ahead contingent claims based on conditional worst case probabilities, given by

$$M_0^t = \beta^t \lambda_t, \quad (2.19)$$

where  $\lambda_t$  is the marginal utility of consumption at time  $t$  by the representative household.

Compared to a standard model of full information and expected utility, the firm problem in (2.18) has two important specific characteristics. The first is that, as described above, unlike the case of expected utility, the firm-level uncertainty that shows up in these state prices does not vanish under diversification. The second concerns the role of experimentation. Under incomplete information but Bayesian decision making, experimentation is valuable because it raises expected utility by improving posterior precision. Here, ambiguity-averse agents also value experimentation since it affects utility by tightening the set of conditional probability considered. Therefore, firms take into account in their problem (2.18) the impact of the level of input on worst-case mean.<sup>21</sup>

We summarize the timing of events within a period  $t$  as follows:

#### 1. Stage 1 : Pre-production stage

- Agents observe the realization of economy-wide shocks (here  $A_t$ ).
- Given forecasts about the idiosyncratic profitability and its associated worst-case scenario, firms hire labor  $H_{l,t}$  and rent capital  $K_{l,t-1}$ . The household supplies labor

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<sup>21</sup>When we present our quantitative results, we assess the contribution of experimentation by comparing our baseline results with those under passive learning, i.e. where there is no active experimentation.

$H_t$  and capital  $K_{t-1}$  and the labor and capital rental markets clear at the wage rate  $W_t$  and capital rental rate  $r_t^K$ .

## 2. Stage 2 : Post-production stage

- Idiosyncratic shocks  $z_{l,t}$  and  $\nu_{l,t}$  realize (but are unobservable) and production takes place.
- Given output and input, firms update estimates about their idiosyncratic profitability and use it to form forecasts for production next period.
- Firms pay out dividends  $D_{l,t}$ . The household makes consumption, investment, and asset purchase decisions ( $C_t$ ,  $I_t$ ,  $B_t$ , and  $\theta_{l,t}$ ).

## 2.5 Log-linearized solution

We solve for the equilibrium law of motion using standard log-linear methods. This is possible for two reasons. First, since the filtering problem firms face is linear, the law of motion of the posterior variance can be characterized analytically. Because the level of inputs has first-order effects on the level of posterior variance, linearization captures the impact of economic activity on confidence. Second, we consider ambiguity about the mean and hence the feedback from confidence to economic activity can be also approximated by linearization. In turn, log-linear decision rules facilitate aggregation because the cross-sectional mean becomes a sufficient statistic for tracking aggregate dynamics.

We log-linearize equilibrium conditions around the steady state based on the worst-case beliefs.<sup>22</sup> Given the equilibrium laws of motion we then characterize the dynamics of the economy under the true DGP. Our solution method extends the one in Ilut and Schneider (2014) by endogenizing the process of ambiguity perceived by the representative household. More substantially, the methodology allows for a tractable aggregation of the endogenous uncertainty faced by heterogeneous firms.

Details on the recursive representation are in Appendix 6.1. Appendix 6.5.2 presents the optimality conditions, which will be a subset of those characterizing the estimated model with additional rigidities introduced in section 4.1. Appendix 6.2 provides a general description of the solution method. Finally, Appendix 6.3 illustrates the log-linearization logic and the feedback between the average level of activity and the cross-sectional average of the worst-case mean by simple expressions for the expected worst-case output and realized output.

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<sup>22</sup>Potential complications arise because the worst-case TFP depends on the level of economic activity. Since the worst-case TFP, in turn, determines the level of economic activity, there could be multiple steady states, i.e. low (high) output and high (low) uncertainty, similar to the analysis in Fajgelbaum et al. (2017). We circumvent this multiplicity by treating the posterior variance of the level of idiosyncratic TFP as a parameter and by focusing on the unique steady state implied that posterior variance.



### 3 Propagation mechanism

In this section we characterize the main properties of the propagation mechanism implied by the endogenous firm-level uncertainty. A crucial part of understanding those dynamics is to explore the way in which the model generates *as if correlated* wedges that respond to the productive *endogenous* inputs chosen in the economy, such as labor and investment. Therefore, these wedges manifest *conditional on any type* of fundamental shock, as long as that shock affects these productive choices. These fundamental shocks can arise in any type of general forms, including standard productivity, demand or monetary policy shocks, as well as more recently proposed sources, such as disturbances in the financial sector, exogenous changes in beliefs, perceived volatility or confidence.

#### 3.1 Co-movement and endogenously correlated wedges

Of particular importance for aggregate dynamics is the implied correlation between the fundamental shock and a labor wedge. This endogenous correlation provides the potential for a wide class of fundamental shocks to produce the basic business cycle pattern of co-movement between hours, consumption and investment, without additional rigidities.

##### *Labor wedge*

The optimal labor tradeoff of equating the marginal cost to the expected marginal benefit under the worst-case belief  $E_t^*$  is given by:

$$H_t^\phi = E_t^*(\lambda_t MPL_t) \quad (3.1)$$

In the standard model, there is no expectation on the right-hand side. As emphasized by Barro and King (1984), there hours and consumption move in opposite direction unless there is a TFP or a preference shock to hours worked in agent's utility (2.14).

Instead, in our model, there can be such co-movement. Suppose that there is a period of low confidence. From the negative wealth effect current consumption is low and marginal utility  $\lambda_t$  is high, so the standard effect would be to see high labor supply as a result. However, because the firm chooses hours *as if* productivity is low, there is a counter substitution incentive for hours to be low. To see how the model generates a countercyclical labor wedge, note that a decrease in hours worked due to an increase in ambiguity, looks, from the perspective of an econometrician, like an increase in the labor income tax. The labor wedge can now be easily explained by implicitly defining the labor tax  $\tau_t^H$  as

$$H_t^\phi = (1 - \tau_t^H)\lambda_t MPL_t \quad (3.2)$$

Using the optimality condition in (3.1), the labor tax is

$$\tau_t^H = 1 - \frac{E_t^*(\lambda_t MPL_t)}{\lambda_t MPL_t} \quad (3.3)$$

Consider first the linear rational expectations case. There the role of firm-level uncertainty disappears and the labor tax in equation (3.3) is constant and equal to zero. To see this, note our timing assumption that labor is chosen after the economy-wide shocks are realized and observed at the beginning of the period. This makes the optimality condition in (3.1) take the usual form of an intratemporal labor decision.<sup>23</sup>

Consider now the econometrician that measures realized  $H_t, C_t$  and  $MPL_t$  in our model. The ratio in equation (3.3) between the expected benefit to working  $\lambda_t MPL_t$  under the worst-case belief compared to the econometrician's measure, which uses the average  $\mu = 0$ , is not equal to one due to the distorted belief. This ratio is affected by standard wealth and substitution effects. Take for example a period of low confidence. On the one hand, since the agent is now more worried about low consumption, the agent's expected marginal utility  $\lambda_t$  is larger than measured by the econometrician's. On the other hand, now the expected marginal product of labor  $MPL_t$  is lower than measured by the econometrician. When the latter substitution effect dominates, the econometrician rationalizes the 'surprisingly low' labor supply by a high labor tax  $\tau_t^H$ .<sup>24</sup>

In turn, periods of low confidence are generated endogenously from a low level of average economic activity, as reflected in the lower cross-sectional average of the worst-case mean, as given by equation (2.17). Therefore, when the substitution effect on the labor choice dominates, the econometrician finds a systematic negative relationship between economic activity and the labor income tax. This relationship is consistent with empirical studies that suggest that in recessions labor falls by more than what can be explained by the marginal rate of substitution between labor and consumption and the measured marginal product of labor

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<sup>23</sup>If we would assume that labor is chosen before the aggregate shocks are realized, there would be a fluctuating labor tax in (3.3) even in the rational expectations model. In that model, the wedge is  $\tau_t^H = 1 - \frac{E_{t-1}(\lambda_t MPL_t)}{\lambda_t MPL_t}$ , where, by the rational expectations assumptions,  $E_{t-1}$  reflects that agents form expectations using the econometrician's data generating process. Crucially, in such a model, the labor wedge  $\tau_t^H$  will not be predictable using information at time  $t-1$ , including the labor choice, such that  $E_{t-1}\tau_t^H = 0$ . In contrast, our model with learning produces predictable, countercyclical, labor wedges.

<sup>24</sup>Given the equilibrium confidence process, which determines the worst-case belief  $E_t^*$ , the economic reasoning behind the effects of distorted beliefs on labor choice has been well developed by existing work, such as Angeletos and La'O (2009, 2013). There they describe the key income and substitution forces through which correlated higher-order beliefs, a form of confidence shocks, show up as labor wedges in a model where hiring occurs under imperfect information on its return. In addition, Angeletos et al. (2014) emphasize the critical role of beliefs being about the short-run rather than the long-run activity in producing stronger substitution effects. In our setup agents learn about the stationary component of firm-level productivity and therefore the equilibrium worst-case belief typically leads to such stronger substitution effects.

(see for example Shimer (2009) and Chari et al. (2007)).

Finally, for an ease of exposition, we have described here the behavior of the labor wedge by ignoring the potential effect of experimentation on the optimal labor choice. This effect may add an additional reason why labor moves ‘excessively’, from the perspective of an observer that only uses equation (3.1) to understand labor movements. In our quantitative model, as discussed later in section 4.4.1, we find that experimentation slightly amplifies the effects of uncertainty on hours worked during the short-run.

#### *Intertemporal consumption wedge*

Uncertainty also affects the consumption-savings decision of the household. This is reflected in the Euler condition for the risk-free asset:

$$1 = \beta R_t E_t^* (\lambda_{t+1} / \lambda_t) \quad (3.4)$$

As with the labor wedge, let us implicitly define an intertemporal consumption wedge:

$$1 = (1 + \tau_t^B) \beta R_t E_t (\lambda_{t+1} / \lambda_t) \quad (3.5)$$

Importantly, this wedge is time varying, since the bond is priced under the uncertainty adjusted distribution,  $E_t^*$ , which differs from the econometrician’s DGP, given by  $E_t$ . By substituting the optimality condition for the interest rate from (3.4), the wedge becomes:

$$1 + \tau_t^B = \frac{E_t^* \lambda_{t+1}}{E_t \lambda_{t+1}} \quad (3.6)$$

Equation (3.6) makes transparent the predictable nature of the wedge. In particular, during low confidence times, the representative household acts *as if* future marginal utility is high. This heightened concern about future resources drives up demand for safe assets and leads to a low interest rate  $R_t$ . However, from the perspective of the econometrician, the measured average marginal utility at  $t + 1$  is not particularly high. To rationalize the low interest rate without observing large changes in the growth rates of marginal utility, the econometrician recovers a high consumption wedge  $\tau_t^B$ , or a high ‘tax’ on consumption. Therefore, the model offers a mechanism to generate movements in the relevant stochastic discount factor that arise endogenously as a countercyclical desire to save in risk-free assets.

#### *Excess return*

Conditional beliefs matter also for the Euler condition for capital:

$$\lambda_t = \beta E_t^* [\lambda_{t+1} R_{t+1}^K]. \quad (3.7)$$

Under our linearized solution, using equation (3.4), we get  $E_t^* R_{t+1}^K = R_t$ , where  $E_t^* R_{t+1}^K$  is the expected return on capital under the worst-case belief. As with the intertemporal consumption wedge, let us define the measured excess return wedge as

$$E_t R_{t+1}^K = R_t(1 + \tau_t^K) \quad (3.8)$$

As with bond pricing, this wedge is time-varying and takes the form

$$1 + \tau_t^K = \frac{E_t R_{t+1}^K}{E_t^* R_{t+1}^K} \quad (3.9)$$

During low confidence times demand for capital is ‘surprisingly low’. This is rationalized by the econometrician, measuring  $R_{t+1}^K$  under the true DGP, as a high ex-post excess return  $R_{t+1}^K - R_t$ , or as a high wedge  $\tau_t^K$  in equation (3.9). In the linearized solution, the excess return, similarly to the labor tax and the discount factor wedge, is inversely proportional to the time-varying confidence. In times of low economic activity, when confidence is low, the measured excess return is high.

Putting together the consumption wedge and the excess return we can characterize the linearized version of the Euler equation for capital in (3.7) as

$$\lambda_t = \frac{(1 + \tau_t^B)}{(1 + \tau_t^K)} \beta E_t[\lambda_{t+1} R_{t+1}^K]. \quad (3.10)$$

Equation (3.10) and the emergence of *both*  $\tau_t^B$  and  $\tau_t^K$  provide cross-equation restrictions that connects our model to three interpretations of shocks to the Euler equations present in the literature. First, it clarifies that the  $\tau_t^B$  wedge does not simply take the form of an ‘as if’ shock to  $\beta$ . If that would be the case, then  $\tau_t^K$  would be zero since the desire to save through a higher  $\beta$  would show up equally in the Euler equations for bonds in (3.4) and capital in (3.7).<sup>25</sup> Second, it clarifies that the friction generates more than just an ‘as if’ tax in the capital market. If that would be the case, then  $\tau_t^B$  would be zero since the desire of the representative agent to save would not be affected.<sup>26</sup> Third, the simultaneous presence of the two wedges relates the friction to a large DSGE literature that uses reduced-form ‘risk-premium’ shocks. Such shocks are introduced as a stochastic preference for risk-free over risky assets, by distorting the Euler equation for bonds but not for capital, which can be interpreted

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<sup>25</sup>See Christiano et al. (2005) and Smets and Wouters (2007) as examples of a large literature of DSGE models that use shocks to  $\beta$ . Recent work, such as Eggertsson and Woodford (2003) and Christiano et al. (2015), also models the heightened desire to save as an independent stochastic shock that is responsible for the economy hitting the zero lower bound on the nominal interest rate.

<sup>26</sup>Quantitative DSGE models typically employ these as if taxes when modeling financial frictions. See for example Gilchrist and Zakrajšek (2011), Christiano et al. (2014) and Del Negro and Schorfheide (2013).

in our model as  $\tau_t^B = \tau_t^K$ .<sup>27</sup>

Therefore, the model predicts that in a recession we, as econometricians, should observe ‘excessively low’ hours worked, at the same time when prices of riskless assets and excess returns for risky assets are ‘excessively inflated’. These correlations arise from any type of shock that moves the economic activity.

## 3.2 Endogenous uncertainty as a parsimonious mechanism

We conclude the description of the model’s qualitative properties by discussing the generality of the proposed economic forces. There are three basic features of uncertainty that were crucial in our proposed mechanism for business cycle dynamics. First, the accumulation of information about relevant profitability prospects occurs through production. Second, the cross-sectional average estimation uncertainty is lower in times when the cross-sectional average production is larger. Third, this state-dependent estimation uncertainty affects consumption and production decisions, including the labor choice. We now discuss alternative modeling specifications that alter some of our specific benchmark choices but still fit within the basic features of uncertainty that matter for our general proposed mechanism.

### *Learning from aggregate market outcomes*

An alternative approach to generate the negative feedback loop between estimation uncertainty and aggregate economic activity is to modify two of our basic features by the following assumptions. First, firms learn about the aggregate-level productivity  $A_t$ . Second, lower aggregate output corresponds to fewer signals available to the firms. This approach of learning from market outcomes is present, in different forms, in the existing macroeconomic literature on endogenous uncertainty, such as Caplin and Leahy (1993), van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), Fajgelbaum et al. (2017) and Saijo (2017).

In a setup with ambiguity like ours, where uncertainty changes the decision maker’s plausible set of conditional means, this alternative approach of learning from market outcomes generates a propagation mechanism for the aggregate dynamics that is qualitatively similar to our benchmark model. The reason is that in both approaches the cross-sectional average estimation uncertainty is countercyclical and that uncertainty affects beliefs about aggregate conditions. Indeed, as discussed in section 2.4, even when ambiguity is solely about the mean of each firm’s productivity, the law of large numbers still preserves an effect of firm-level uncertainty on the worst-case beliefs of the cross-sectional average productivity.

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<sup>27</sup>Reduced-form risk premium shocks have typically emerged as a key business cycle driver in quantitative DSGE models, starting with Smets and Wouters (2007). See Gust et al. (2017) for a recent contribution emphasizing the quantitative role of these shocks. See Fisher (2015) for an interpretation of these shocks as time-varying preference for liquidity.

We highlight the robust qualitative features of the feedback between uncertainty and activity in a stylized representative firm RBC model without capital. In this simple model we make two key assumptions: labor is chosen before productivity is known and there is a negative relationship between current ambiguity and past labor choice. Both of these features arise endogenously in our benchmark model or in a model of learning from aggregate outcomes.

We present the details of this stylized model in Appendix 6.4. There we allow for two sources of macro disturbances, an iid economy-wide TFP shock and a persistent government spending shock. The linearity of the model allows us to solve it in closed-form and show the main qualitative features that are common to our benchmark model of endogenous uncertainty. First, endogenous confidence leads to an AR(2) term in the law of motion for hours worked that can generate hump-shaped and persistent dynamics. Second, both consumption and hours can rise after an increase in government spending. Third, the model can generate predictable countercyclical wedges, driven by the past hours worked, on labor supply, risk-free and risky assets. Fourth, policy interventions are affected by the endogenous confidence process. In particular, the government spending multiplier is now larger.

While qualitatively similar to learning from aggregate market outcomes in its implications for aggregate dynamics, the friction present in our benchmark model, namely learning about firm-level profitability, has also some qualitatively different properties. First, the competitive equilibrium of our economy is constrained Pareto optimal. Indeed, in this world there are no information externalities since learning occurs at the individual firm level and not from observing the aggregate economy. This stands in contrast to the case of learning from aggregate market outcomes, where an individual firm does not take into account the positive externality of generating signals that are useful for the rest of the economy. Thus, even if policy interventions affect the aggregate dynamics similarly in the two cases, the welfare properties are different. For example, the increased economic activity, and the associated increase in the signal-to-noise ratio, produced by a government spending increase is not welfare increasing in our model. Second, extending the sources of imperfect information to firm-level shock offers a new way of disciplining endogenous uncertainty process through micro data. These include, as we will discuss in our quantitative model, firm-level technological or informational parameters.

#### *Uncertainty as risk only*

The third ingredient of our mechanism is that uncertainty comprises both risk and ambiguity. Consider now a version of the model in which there is no ambiguity. Since all optimality conditions have been log-linearized the countercyclical uncertainty does not feed back into economic activity. Indeed, countercyclical perceived risk at the firm level may matter for the aggregate dynamics only insofar as it affects average production decisions through non-linear policy functions.

On the methodological side, a model where uncertainty is only risk requires non-linear solution methods and keeping track of the time-varying distribution of firms.<sup>28</sup> In contrast, in our model, even with linear policy functions the endogenous countercyclical firm-level uncertainty matters. The reason is that uncertainty also includes ambiguity, an effect that, as discussed in section 2.4, is first-order and aggregates up linearly by the LLN.

In terms of specific business cycle implications, a model with risk and non-linear policy functions shares similarities with our findings. While the details on non-linearities differ, a typical finding in the literature is that the higher risk in recessions may lead to a contraction in average investment.<sup>29</sup> Whether a model with risk only can generate co-movement between consumption, hours and investment then depends on the strength of the implied productivity or labor ‘wedges’.<sup>30</sup>

Therefore, ambiguity offers a novel theoretical channel through which firm-level uncertainty shapes aggregate outcomes. Together with the learning effect from activity to uncertainty, it provides a new laboratory for both a transparent and quantitative evaluation of the role of endogenous firm-level uncertainty as a propagation mechanism.

## 4 Quantitative analysis

We now bring our endogenous uncertainty mechanism to the data in order to quantify the potential of the proposed information friction as a propagation mechanism and contrast it to other frictions. Our analysis consists of four steps. First, we embed the friction into a standard medium-scale business cycle model by allowing for an array of real and nominal rigidities. Second, we employ an estimation procedure that focuses squarely on propagation. Since our friction predicts that we should observe regular patterns of co-movement and correlated wedges *conditional* on different types of shocks, our estimation consists of matching the model-implied and empirical impulse responses for shocks identified by Structural Vector Autoregressive models in the literature. Third, we run monetary and fiscal policy experiments to evaluate the impact of our friction on policies. Fourth, we use observable dispersion of beliefs to externally test the model’s implications.

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<sup>28</sup>See Terry (2017) for an analysis of approaches to solve heterogeneous firm models with aggregate shocks.

<sup>29</sup>This may work through an extensive margin, from a real option argument as in Bloom (2009), or an intensive margin, through decreasing returns to scale as in Senga (2015).

<sup>30</sup>One specific channel is to assume that labor is chosen before a cash flow shock is realized, as in some models of financial frictions. There a higher idiosyncratic uncertainty, either exogenous (as in Arellano et al. (2012)) or endogenous (as in Gourio (2014)), about that cash flow realization, may lead to a labor wedge. A second more general channel in these types of heterogeneous firm models with non-linearities is the implied endogenous TFP fluctuations arising from mis-allocation.

## 4.1 A medium-scale DSGE model

We add several standard features to the estimated model. The production function with capital utilization is

$$F_{l,t} = (U_{l,t}K_{l,t-1})^\alpha (\gamma^t H_{l,t})^{1-\alpha}$$

where  $\gamma$  is the deterministic growth rate of the economy and  $a(U_{l,t})K_{l,t-1}$  is an utilization cost that reduces dividends in equation (2.5).<sup>31</sup>

We modify the representative household's utility (2.14) to allow for habit persistence in consumption:

$$U_t(C; s^t) = \ln(C_t - bC_{t-1}) - \frac{H_t^{1+\phi}}{1+\phi} + \beta \min_{\mu_{l,t} \in [-a_{l,t}, a_{l,t}], \forall l} E^\mu[U_{t+1}(C; s^t, s_{t+1})],$$

where  $b > 0$  is a parameter. We also introduce an investment adjustment cost:

$$K_t = (1 - \delta)K_{t-1} + \left\{ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - \gamma \right)^2 \right\} I_t, \quad (4.1)$$

where  $\kappa > 0$  is a parameter. For nominal rigidities we consider standard Calvo-type price and wage stickiness, along with monopolistic competition.<sup>32</sup>

We follow Bernanke et al. (1999) and introduce entrepreneurs that purchase capital from households and use it to produce output. The purchases of capital are financed by two sources: their own net worth and borrowing from financial intermediaries. The financial intermediaries provide external finance to entrepreneurs using funds obtained from households.

The agency problem between entrepreneurs and financial intermediaries gives rise to an external finance premium. We introduce a financial shock,  $\Delta_t^K$ , in the form of a time-varying difference between the financial intermediaries' revenue and its opportunity cost of its funds (the risk-free return). We assume  $\Delta_t^K$  follows an AR(1) process:

$$\ln \Delta_t^K = \rho_\Delta \ln \Delta_{t-1}^K + \epsilon_{\Delta,t},$$

where the innovation  $\epsilon_{\Delta,t}$  is iid Gaussian with a standard deviation  $\sigma_\Delta$ . An increase in  $\Delta_t^K$

<sup>31</sup>We specify:  $a(U) = 0.5\chi_1\chi_2U^2 + \chi_2(1 - \chi_1)U + \chi_2(0.5\chi_1 - 1)$ , where  $\chi_1$  and  $\chi_2$  are parameters. We set  $\chi_2$  so that the steady-state utilization is one. The cost  $a(U)$  is increasing in utilization and  $\chi_1$  determines the degree of the convexity of utilization costs. In a linearized equilibrium, the dynamics are controlled by the  $\chi_1$ .

<sup>32</sup>We follow Bernanke et al. (1999) and assume that the monopolistic competition happens at the "retail" level. Retailers purchase output from firms in a perfectly competitive market, differentiate them, and sell them to final-goods producers, who aggregate retail goods using the conventional CES aggregator. The retailers are subject to the Calvo friction and thus can adjust their prices in a given period with probability  $1 - \xi_p$ . To introduce sticky wages, we assume that households supply differentiated labor services to the labor packer with a CES technology who sells the aggregated labor service to firms. Households can only adjust their wages in a given period with probability  $1 - \xi_w$ .



raises the credit spread (the difference between the loan rate to entrepreneurs and the risk-free rate) and drives up the cost of external finance. The interpretation and identification of this financial shock follows the standard literature, along the lines of Gilchrist and Zakrajšek (2012). It could reflect changes in costs of financial intermediation that are caused by disruptions in the financial system or variations in the households' attitudes towards risky assets due to, for instance, fluctuations in liquidity conditions in the secondary market for these securities. It could also originate from a reduction in the supply of credit that are caused by a deterioration in the balance-sheet condition of the financial intermediaries.<sup>33</sup> Appendix 6.5.1 provides a complete exposition of the financial friction and the financial shock.

The central bank follows a Taylor-type rule. We consider a general form and allow the monetary authority to respond to current and lagged endogenous variables:

$$\hat{R}_t = \sum_{i=1}^2 \rho_R^i \hat{R}_{t-i} + \sum_{i=0}^2 \phi_\pi^i \hat{\pi}_{t-i} + \sum_{i=0}^2 \phi_Y^i \Delta \hat{Y}_{t-i} + \epsilon_{R,t}, \quad \epsilon_{R,t} \sim N(0, \sigma_R^2),$$

where  $\rho_R^i$ ,  $\phi_\pi^i$ , and  $\phi_Y^i$  are parameters and  $\epsilon_{R,t}$  is a monetary policy shock.

## 4.2 A structural VAR analysis

The starting point of our empirical investigation is a structural VAR (SVAR) analysis of U.S. quarterly macroeconomic data over the sample period 1980Q1–2008Q3. The sample starts after the Volcker appointment to avoid parameter instabilities regarding monetary policy. Similarly, we trim the observation after 2008Q4 in order to avoid complications arising from the zero lower bound. The three structural shocks — technology, financial and monetary policy shocks — are recursively identified. Our two-lag VAR includes the following variables: (1) log-difference of utilization-adjusted TFP from Fernald (2014), (2) the difference of (min-max) range of one quarter ahead forecasts for Q/Q real GDP growth from the Survey of Professional Forecasters (SPF), (3) log-difference of real GDP, (4) log hours worked, (5) log-difference of real investment, (6) log-difference of real consumption, (7) log-difference of real wages, (8) log GDP deflator inflation, (9) credit spread (GZ spread) from Gilchrist and Zakrajšek (2012), (10) return on assets<sup>34</sup>, and (11) log federal funds rate.<sup>35</sup> The identifying assumptions implied by the ordering are (a) technology shocks affect all variables instantaneously and that utilization-adjusted TFP does not respond to innovations to other shocks in the current

<sup>33</sup>See Gilchrist and Zakrajšek (2011), Christiano et al. (2014), Del Negro and Schorfheide (2013) and Lindé et al. (2016) for recent DSGE models that incorporate variants of this financial shock.

<sup>34</sup>We use the return on assets of the U.S. corporate sector calculated from Compustat data by Gilchrist and Zakrajšek (2012).

<sup>35</sup>For variables that enter in (log-)differences we cumulate the impulse responses so that they are expressed in levels.

period, (b) financial shocks (shocks to the credit spread) move all variables except for the return on assets and the fed rate with a lag, and (c) monetary policy shocks affect other variables with a lag. We modify the timing of the quantitative model so that it is consistent with the identifying assumptions above.

Table 1: Variance decomposition at business cycle frequencies

	Technology	Financial	Monetary policy
Output	12.0 (2.3, 28.7)	21.5 (3.8, 40.5)	3.0 (0.5, 9.1)
Hours	6.4 (0.2, 23.9)	35.3 (8.1, 51.7)	3.4 (0.2, 10.8)
Investment	12.2 (1.9, 31.3)	25.5 (4.4, 44.6)	3.3 (0.2, 9.8)
Consumption	7.4 (1.0, 22.9)	10.5 (1.2, 31.0)	1.3 (0.4, 7.3)
Real wages	13.8 (3.4, 31.7)	15.3 (3.1, 35.4)	2.7 (0.2, 8.2)
Inflation	2.1 (0.2, 17.7)	1.4 (0.2, 28.7)	0.8 (0.1, 6.8)
Fed rate	1.1 (0.1, 16.7)	23.1 (4.3, 45.5)	7.0 (1.2, 12.4)

*Notes:* We report the percentage variance in the business cycle frequencies (6–32 quarters) due to the indicated shocks. Numbers in parentheses are the 95 percent intervals. All variables are in log-levels.

Table 1 reports the percentage of variance for each endogenous variable at the business cycle frequency that can be explained by the identified shocks. Financial shocks account for a sizable fraction of fluctuations in the macro quantities. For example, the shock can explain 22 and 35 percent of the business cycle variation in output and hours worked, respectively. The other two shocks also explain a nontrivial amount of fluctuations but are significantly less important. For example, technology and monetary policy shocks account for 12 and 3 percent of output fluctuations, respectively. Finally, all three identified shocks account for a negligible amount of inflation. In particular, the financial shock, which explains a substantial fraction of movements in real quantities, explains only 1.4 percent of inflation. As pointed out by Angeletos (2017), this disconnect between inflation and quantity fluctuations suggests that the data prefers a propagation mechanism that does not rely on nominal rigidities.

### 4.3 Bayesian impulse response matching estimation

We fix a small number of parameters before the estimation. The growth rate of technology  $\gamma$ , the discount factor  $\beta$ , the depreciation rate of capital  $\delta$ , and the share of government

spending to output  $\bar{g}$  are set to 1.004, 0.998, 0.025, and 0.2 respectively. We set  $\theta$ ,  $\theta_p$ ,  $\theta_w$  to 11, which imply steady-state firm-level markups, price markups, and wage markups of 10%. The survival rate of the entrepreneurs is set to  $\zeta = 0.98$  and the steady-state capital to net worth ratio is set to 1.7, which are in line with the values used in Bernanke et al. (1999).

The remaining set of parameters is estimated using a Bayesian version of a impulse-response-matching method, developed by Christiano et al. (2010). The description of the methodology is contained in Appendix 6.5.3.

We conduct two main estimation experiments. In the first one, we estimate our model using only the impulse responses to the financial shock. This shock is particularly informative for our objective for two reasons. First, it is quantitatively important, as it accounts for a significant fraction of business cycle variation. Second, the shock is characterized by cross-equation restrictions, in the form of positive co-movement of aggregate variables as well as correlated wedges, that provide stark identification of the underlying propagation mechanisms.

In the second experiment, we estimate the model using impulse responses to all three identified shocks. This allows us to examine the quantitative robustness of the conclusion from the first experiment, explore the implications of endogenous uncertainty for other structural shocks, and more generally evaluate the role played by the additional cross-equations restrictions in the identification of the model.

For both experiments, we stack the current and 19 lagged values of impulse response functions from 9 of the VAR variables (all variables except SPF dispersion and return on assets) in the vector of responses to be matched. As additional discipline coming from the empirical cross-equation restrictions, we also incorporate the responses of labor and consumption wedges and excess return implicitly computed from the SVAR, using the log-linearized first-order conditions from (3.2), (3.5), and (3.8). To calculate these wedges from the data, we need to take stand on some parameter values. We assume  $\phi = 0.5$  and  $b = 0$ . When we calculate the wedges implied by the models, we use the same log-linearized conditions and parameter values and the expectations are computed under the econometrician's DGP.<sup>36</sup> Thus, in computing the wedges, the data and the model are treated symmetrically.

We use SPF dispersion as an external model validation. As in Ilut and Schneider (2014), we relate the set of forecasts about real GDP growth in the model to the observed dispersion of forecasts in the data. While the ambiguity model produces such a set, the RE does not since since the set of forecasts collapses by assumption in that case to a singleton. Excluding SPF dispersion from the estimation criterion allows us to keep the number of observables between

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<sup>36</sup>To be precise, we use the following equations to calculate the wedges:

$$\hat{\tau}_t^H = -(1 + \phi)\hat{H}_t - \hat{C}_t + \hat{Y}_t; \hat{\tau}_t^B = -\hat{C}_t + E_t\hat{C}_{t+1} - \hat{R}_t + E_t\hat{\pi}_{t+1}; \hat{\tau}_t^K = E_t\hat{R}_{t+1}^K - \hat{R}_t,$$

where  $\hat{R}_{t+1}^K$  is measured using return on assets.

our model and its RE counterpart the same and thus facilitates the comparison between the two models. Nevertheless, when we report the estimated impulse response from our model with ambiguity, we plot the implied range of growth forecasts against that from the SPF.

Table 2: Estimated parameters: preference and technology

		Prior			Single shock		All shocks	
		Type	Mean	Std	Ambiguity	RE	Ambiguity	RE
$\alpha$	Capital share	B	0.3	0.02	0.51 (0.017)	0.34 (0.019)	0.30 (0.007)	0.31 (0.010)
$\phi$	Inv. Frisch elasticity	G	0.5	0.25	0.006 (0.004)	1.26 (0.260)	0.003 (0.002)	0.002 (0.001)
$\chi_1$	Utilization cost	IG	0.01	0.25	0.015 (0.005)	1.14 (0.408)	0.005 (0.001)	0.003 (0.001)
$b$	Consumption habit	B	0.4	0.05	0	0.38 (0.043)	<b>0.78</b> (0.016)	<b>0.83</b> (0.016)
$\kappa$	Investment adj. cost	G	0.5	0.2	0	0.21 (0.056)	<b>0.22</b> (0.009)	<b>0.73</b> (0.089)
$\frac{1}{1-\xi_p}$	Avg. freq. of price adjustment	G	2	0.3	1.0001	7.36 (0.856)	<b>2.67</b> (0.034)	<b>4.51</b> (0.223)
$\frac{1}{1-\xi_w}$	Avg. freq. of wage adjustment	G	2	0.3	1.0001	5.38 (0.410)	<b>1.76</b> (0.031)	<b>3.42</b> (0.189)
$\sigma_\omega$	Std. of entrepreneur shock	G	0.5	0.15	0.47 (0.010)	0.67 (0.046)	0.20 (0.003)	0.17 (0.010)
$\mu$	Monitoring cost	B	0.1	0.03	0.0003 (0.0001)	0.07 (0.001)	0.09 (0.021)	0.07 (0.011)
$\Delta^K$	SS financial shock	B	0.015	0.01	0.014 (0.0001)	0.008 (0.002)	0.013 (0.0001)	0.001 (0.0002)
$\rho_z$	Idiosyncratic shock	B	0.6	0.2	0.65 (0.017)	–	0.92 (0.005)	–
$\sigma_z$	Idiosyncratic shock	B	0.4	0.03	0.71 (0.020)	–	0.56 (0.008)	–
$0.5\eta$	Entropy constraint	B	0.5	0.2	0.99 (0.006)	0	0.99 (0.011)	0
$\bar{\Sigma}$	SS posterior variance	G	0.1	0.02	0.09 (0.018)	–	0.09 (0.022)	–

Notes: See notes from Table 3.

Table 2 and 3 report the prior distributions. Since we use standard choices for priors whenever possible, our discussion focuses on the parameters that affect the strength of the feedback loop between economic activity and uncertainty, which are determined by three factors. The first factor is the variability of inputs which is determined by the elasticities of capital utilization and labor supply.  $\chi_1$ , which controls the elasticity of utilization, is centered around 0.01, where lower values indicate more elastic utilization<sup>37</sup>, while the inverse Frisch elasticity  $\phi$  is centered around 0.5. Second, the parameters that are related to the firm-level

<sup>37</sup>The choice of the prior mean is motivated by Christiano et al. (2005), who use  $\chi_1 = 0.01$ .

Table 3: Estimated parameters: monetary policy and structural shocks

		Type	Prior		Single shock		All shocks	
			Mean	Std	Ambiguity	RE	Ambiguity	RE
$\rho_R^1$	Interest smoothing	B	0.4	0.2	0.80	0.23	0.96	0.67
					(0.113)	(0.147)	(0.025)	(0.086)
$\rho_R^2$	Interest smoothing	B	0.4	0.2	0.44	0.07	0.16	0.49
					(0.152)	(0.060)	(0.011)	(0.086)
$\phi_\pi^0$	Inflation response	N	1	0.2	1.43	1.01	1.76	1.02
					(0.173)	(0.158)	(0.029)	(0.193)
$\phi_\pi^1$	Inflation response	N	1	0.2	1.19	0.84	0.90	0.81
					(0.221)	(0.158)	(0.032)	(0.162)
$\phi_\pi^2$	Inflation response	N	1	0.2	0.94	0.69	1.09	0.60
					(0.179)	(0.113)	(0.020)	(0.140)
$\phi_Y^0$	Output response	N	0	0.1	-0.25	0.21	0.35	0.27
					(0.050)	(0.056)	(0.015)	(0.043)
$\phi_Y^1$	Output response	N	0	0.1	-0.00	-0.11	0.25	0.21
					(0.076)	(0.099)	(0.026)	(0.059)
$\phi_Y^2$	Output response	N	0	0.1	0.00	-0.48	-0.17	-0.21
					(0.089)	(0.057)	(0.028)	(0.058)
$\rho_\Delta$	Financial	B	0.6	0.2	0.99	0.87	0.73	0.81
					(0.001)	(0.020)	(0.004)	(0.033)
$100\sigma_\Delta$	Financial	IG	1	1	2.71	4.27	3.96	46.97
					(0.143)	(1.071)	(0.036)	(6.737)
$\rho_A$	Technology	B	0.6	0.2	–	–	0.99	0.99
							(0.0001)	(0.0001)
$100\sigma_A$	Technology	IG	1	1	–	–	0.32	0.35
							(0.005)	(0.007)
$100\sigma_R$	Monetary policy	IG	0.1	1	–	–	0.09	0.11
							(0.021)	(0.004)
Log marginal likelihood					<b>-576</b>	<b>-623</b>	<b>-1487</b>	<b>-1714</b>

*Notes:* ‘Single shock’ refers to the posterior modes of the estimation using only the financial shock and ‘All shocks’ refers to the posterior modes from the estimation using all three shocks. ‘Ambiguity’ corresponds to the baseline model with endogenous uncertainty and ‘RE’ corresponds to its rational expectations version.  $B$  refers to the Beta distribution,  $N$  to the Normal distribution,  $G$  to the Gamma distribution,  $IG$  to the Inverse-gamma distribution. Posterior standard deviations are in parentheses and are obtained from draws using the random-walk Metropolis-Hasting algorithm. The marginal likelihood is calculated using Geweke’s modified hamonic mean estimator.

processes control how changes in inputs translate to changes in posterior variance. Given the wide range of estimates for the persistence of the idiosyncratic shocks, we set a relatively diffuse prior for  $\rho_z$  centered around 0.6. Guided by the establishments-level evidence by Bloom et al. (2018), we set the prior mean of the innovation  $\sigma_z$  to be 0.4. David et al. (2015) estimate the posterior variance of a firm-specific shock to be around 8–13%. We set the prior mean for the posterior variance at the zero-risk steady state  $\bar{\Sigma}$  to 10%.<sup>38</sup> Finally, the size of the entropy constraint  $\eta$  determines how changes in the posterior standard deviation translate into changes in confidence. Ilut and Schneider (2014) argue that a reasonable upper bound for  $\eta$  is 2, based on the view that agents’ ambiguity should not be “too large”, in a statistical sense, compared to the variability of the data. We re-parametrize the parameter and estimate  $0.5\eta$ , for which we set a Beta prior.<sup>39</sup>

## 4.4 Results

### 4.4.1 Estimation using impulse responses for the financial shock

Our first experiment is to estimate the model using only the impulse response to the financial shock. To highlight the properties of our endogenous uncertainty mechanism, we shut down standard rigidities such as consumption habit, investment adjustment cost, sticky prices and wages. We also compare our estimated model with the standard RE model in which we allow all the features, except ambiguity, presented in section 4.1.

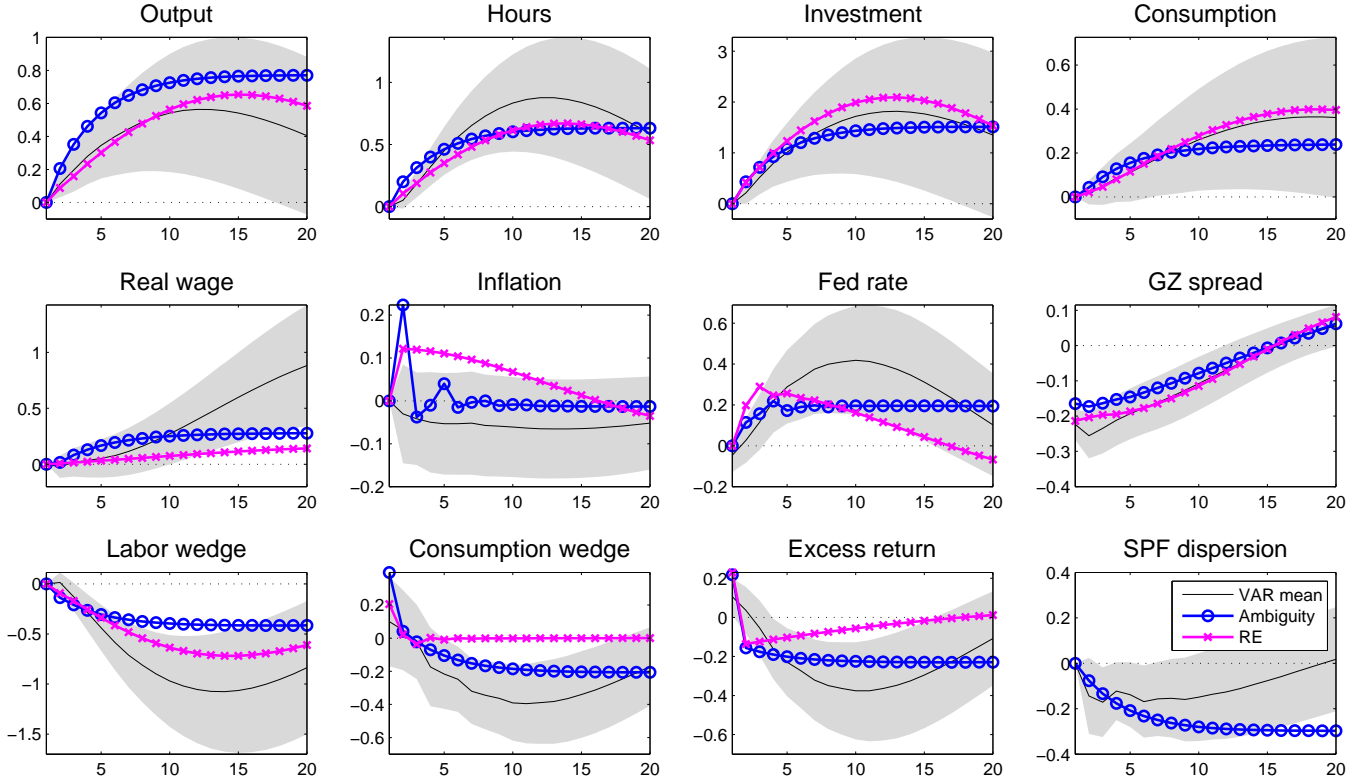
Figure 1 reports the VAR mean impulse responses (labeled ‘VAR mean’) as well as the estimated impulse responses from our model (labeled ‘Ambiguity’) and from the RE model (labeled ‘RE’) to a one-standard deviation financial shock. Columns labeled ‘Single shock’ in Table 2 and 3 report the posteriors. According to the VAR, an expansionary financial shock reduces the credit spread and raises output, hours, investment and consumption in a hump-shaped manner. The real wages and the federal funds rate rise, but inflation does not move, translating into an increase in the real interest rate (Figure 2).<sup>40</sup> Finally, all the three wedges - labor, consumption and the excess return - as well as the forecast dispersion fall.

<sup>38</sup>We re-parameterize the model so that we take the worst-case steady state posterior variance  $\bar{\Sigma}^0$  of idiosyncratic TFP as a parameter. This posterior variance, together with  $\rho_z$  and  $\sigma_z$ , will pin down the standard deviation of the unit-specific shock  $\sigma_\nu$ . The zero-risk steady state is the ergodic steady state of the economy where optimality conditions take into account uncertainty and the data is generated under the econometrician’s DGP. Appendix 6.2 provides additional details.

<sup>39</sup>The priors for the standard deviation of a shock to the entrepreneurs  $\sigma_\omega$  and the monitoring cost  $\mu$  are centered around 0.5 and 0.1, respectively, in line with the values used in Bernanke et al. (1999). We set the prior mean of the steady-state financial shock  $\Delta^K$  to 0.015, motivated by the finding by Phillipon (2015) that financial intermediation costs around 1.5 percent of intermediated assets.

<sup>40</sup>To calculate the real interest rate  $i_t$  from the VAR, we simply compute  $i_t = R_t - E_t\pi_{t+1}$ , where  $R_t$  is the impulse response function for the federal funds rate at period  $t$  and  $E_t\pi_{t+1}$  is the impulse response function for inflation at period  $t + 1$ . The real interest rates from the models are calculated in an analogous manner.

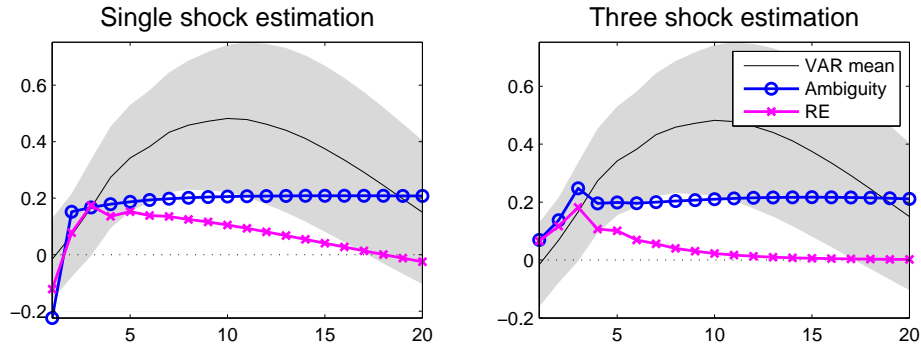
Figure 1: Responses to a financial shock (single shock estimation)



*Notes:* The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue circled lines are the impulse responses from the baseline model with ambiguity but without real and nominal rigidities. The purple lines are the impulse responses from the standard RE model featuring real and nominal rigidities. Both impulse responses are estimated using only the VAR response to the financial shock. The responses of output, hours, investment, consumption and real wages are in percentage deviations from the steady states while inflation, fed rate, GZ spread and excess return are in annual percentage points. The rest are in quarterly percentage points.

Our model with endogenous ambiguity matches the VAR response well. First, our model generates persistent and hump-shaped dynamics as well as co-movement in real quantities. This property is due solely to the endogenous uncertainty mechanism. To see this, in Figure 3 we calculate the responses of real quantities when we turn off ambiguity (set the entropy constraint  $\eta$  to 0) and re-estimate the model. In sharp contrast to the baseline model, output, hours, and investment all rise initially and then monotonically decrease while consumption declines, consistent with the Barro and King (1984) logic. Second, our model generates the increase in real wages because of the rise in confidence. In standard models, absent other forces like countercyclical markups, an increase in labor supply would reduce real wages due to the

Figure 2: The response of the implied real interest rate to a financial shock



*Notes:* The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue circled lines are the impulse responses from the baseline model with ambiguity. The purple lines are the impulse responses from the standard RE model. The left panel is based on the estimation using only the VAR response to the financial shock and the right panel is based on the estimation using the responses to the VAR responses to all three structural shocks (technology, financial and monetary policy). The unit is in annual percentage points.

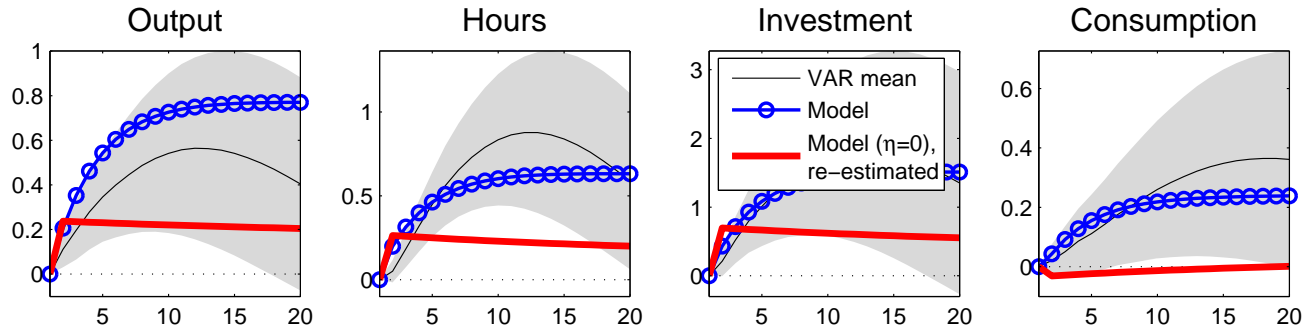
declining marginal product of labor. Third, our model replicates the dynamics of inflation (except for the initial period) and of the nominal interest rate and hence of the real interest rate. Fourth, as a result of these successes, the labor and consumption wedges as well as the excess return fall as in the data, although the model slightly understates the reduction in the labor wedge. Fifth, although not directly targeted in the estimation, the model implies a decline in the forecast range that are in line with the SPF. Finally, in our model agents internalize the effect of their input choices on the evolution of confidence. In Figure 10 in the Appendix, we evaluate the contribution of this experimentation motive by assuming that agents do not internalize this effect (passive learning). We find that experimentation slightly amplifies the responses of output and hours but the main features of the two learning assumptions are virtually identical.

Consider now the RE model. The model is able to generate a persistent rise in output, hours, investment and consumption. This is largely due to the nominal rigidities, where at the posterior mode prices and wages are adjusted roughly every 7 and 5 quarters, respectively, and to a lesser extent due to real rigidities, where at the posterior mode consumption habit  $b = 0.38$  and the investment adjustment cost  $\kappa = 0.21$ . The RE model, however, cannot match several implications for prices. First, the model overpredicts inflation for several periods after the shock. Second, because of the high degree of wage stickiness and since the model generates higher inflation, the model understates the rise in real wages. Third, the left panel of Figure 2 shows that the model underpredicts the real interest rate in the medium run.

To understand the real rate dynamics, consider a standard Euler equation for risk-free assets. In a first-order approximation, the Euler equation implies that expected consumption



Figure 3: Responses to a financial shock: effect of confidence



*Notes:* The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence bands. The blue circled lines are the impulse responses from the baseline model with ambiguity, estimated using only the VAR response to the financial shock. The red solid lines are the impulse responses when we turn off the effect of confidence by setting  $\eta = 0$  and re-estimate the baseline model. In both versions, there is no real or nominal rigidity. Responses are in percentage deviations from the steady states.

growth is equal to the real interest rate. This relationship continues to hold with consumption habit as long as it is moderate. Now consider the dynamics of consumption. Both in the VAR and in the model the consumption growth slows down in the medium run. The Euler equation implies that this should lead to a lower real interest rate, while in the data the interest rate remains persistently high. It is also now clear that our model with ambiguity is able to break this counterfactual link between consumption growth and real interest rate through lowering the effective stochastic discount factor in the Euler equation, manifested as a reduction in the consumption wedge. Finally, the RE model fails to generate a decline in the consumption wedge due to the aforementioned implication of the Euler equation. It also fails to predict a persistent drop in the excess return; instead, in the RE model the excess return traces the financial shock process and hence its fall is transitory.

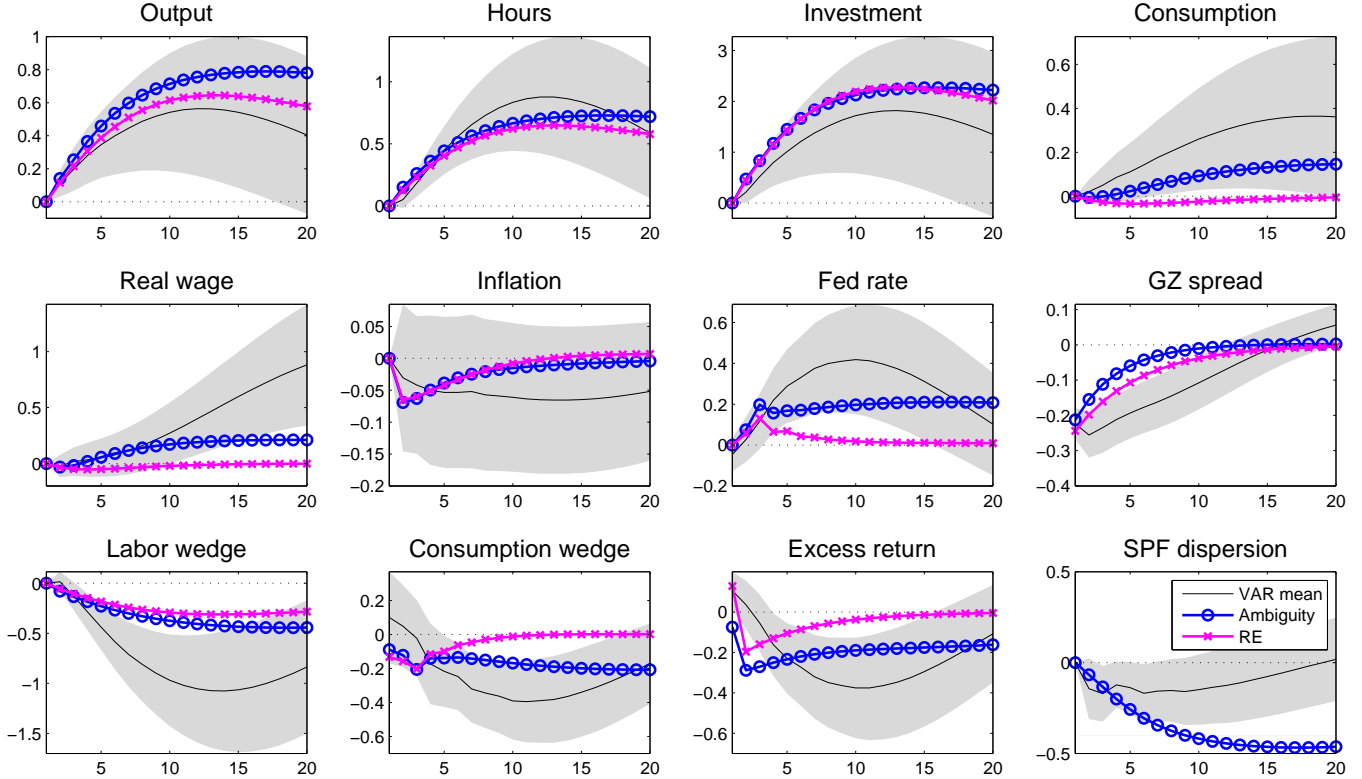
To summarize, our endogenous uncertainty mechanism allows us to successfully replicate the dynamics of real quantities, prices and wedges as well as the dispersion in survey forecasts. In contrast, the RE model can match the dynamics of real quantities but it comes at the expense of counterfactual implications for prices. The RE model also fails to capture the reduction in the consumption wedge and the hump-shaped decline in the excess return. As a result, the data favors our model with ambiguity over the RE model: the marginal likelihood of our model is  $(-576 - (-623)) = 47$  log points higher than the RE model (Table 3).

#### 4.4.2 Estimation using impulse responses for all three shocks

Our second experiment is to estimate the model using all three structural shocks. As in the first experiment, we estimate both the ambiguity model and the RE model. In order to produce real

effects of monetary policy shocks, we incorporate nominal rigidities (sticky prices and wages) and for symmetry also real rigidities (consumption habit and investment adjustment cost) into our ambiguity model. This allows us to ask to what extent our propagation mechanism quantitatively replaces standard rigidities used in medium-scale DSGE models with several structural shocks.

Figure 4: Responses to a financial shock (three shock estimation)



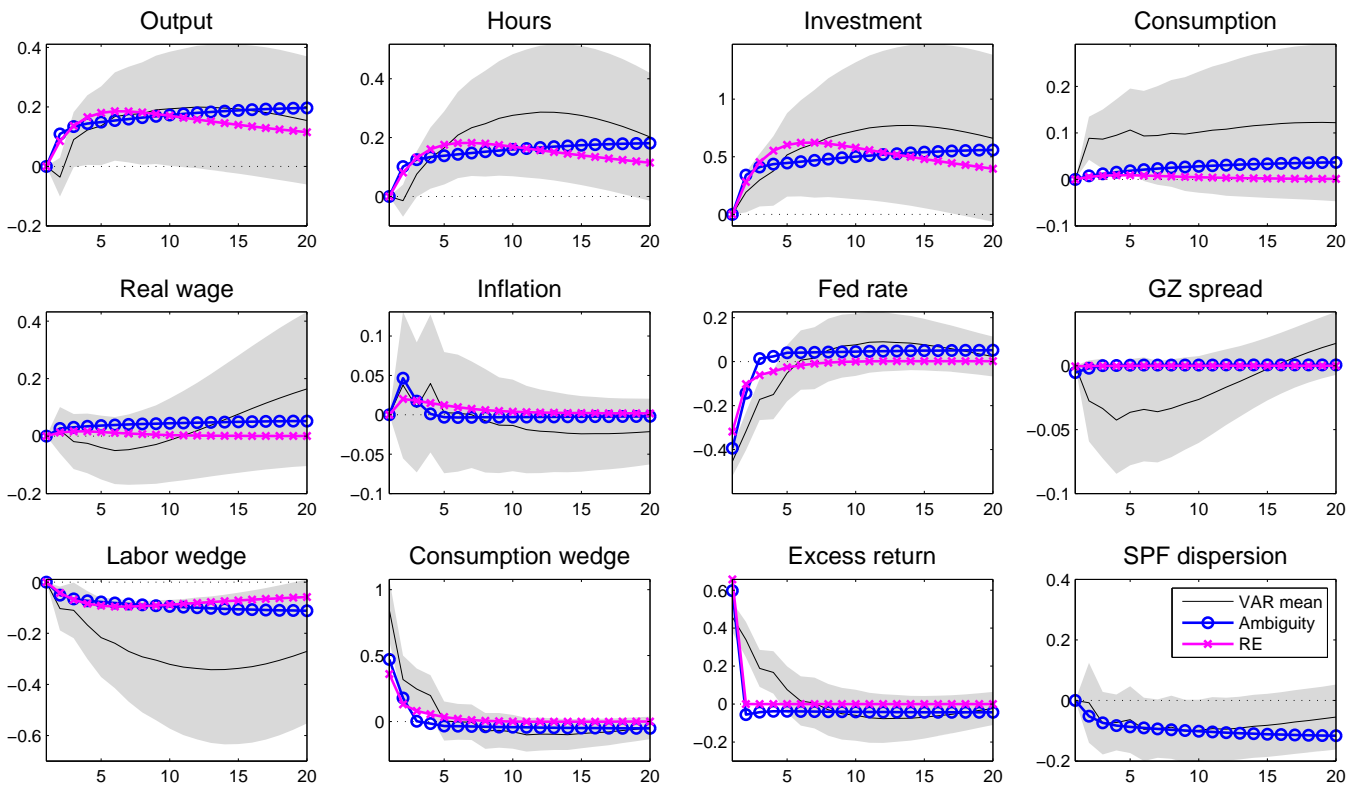
*Notes:* The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence bands. The blue circled lines are the impulse responses from the baseline model with ambiguity and the purple lines are the impulse responses from the standard RE model. Both impulse responses are estimated using the VAR responses to all three structural shocks (technology, financial and monetary policy). The responses of output, hours, investment, consumption and real wages are in percentage deviations from the steady states while inflation, fed rate, GZ spread and excess return are in annual percentage points. The rest are in quarterly percentage points.

Columns labeled ‘All shocks’ in Table 2 and 3 report the posteriors. We begin by comparing the impulse responses for a financial shock in our model and the RE model (Figure 4). First, note that our model, as in the single shock estimation, is broadly successful in replicating the impulse response to the financial shock. The three main differences compared to the

single shock estimation are that: (i) there is no longer the initial spike in inflation thanks to sticky prices, (ii) the consumption increase is smaller due to habit, and (iii) the model slightly overstates the reduction in dispersion.

In contrast to our model, the RE model fails to replicate the key features of the data. In particular, the model no longer generates co-movement between consumption and other real quantities such as output and hours. In addition, the model significantly understates the rise in nominal and real interest rates (right panel of Figure 2). Instead, consumption and the risk-free rate barely move and are negatively correlated.

Figure 5: Responses to a monetary policy shock

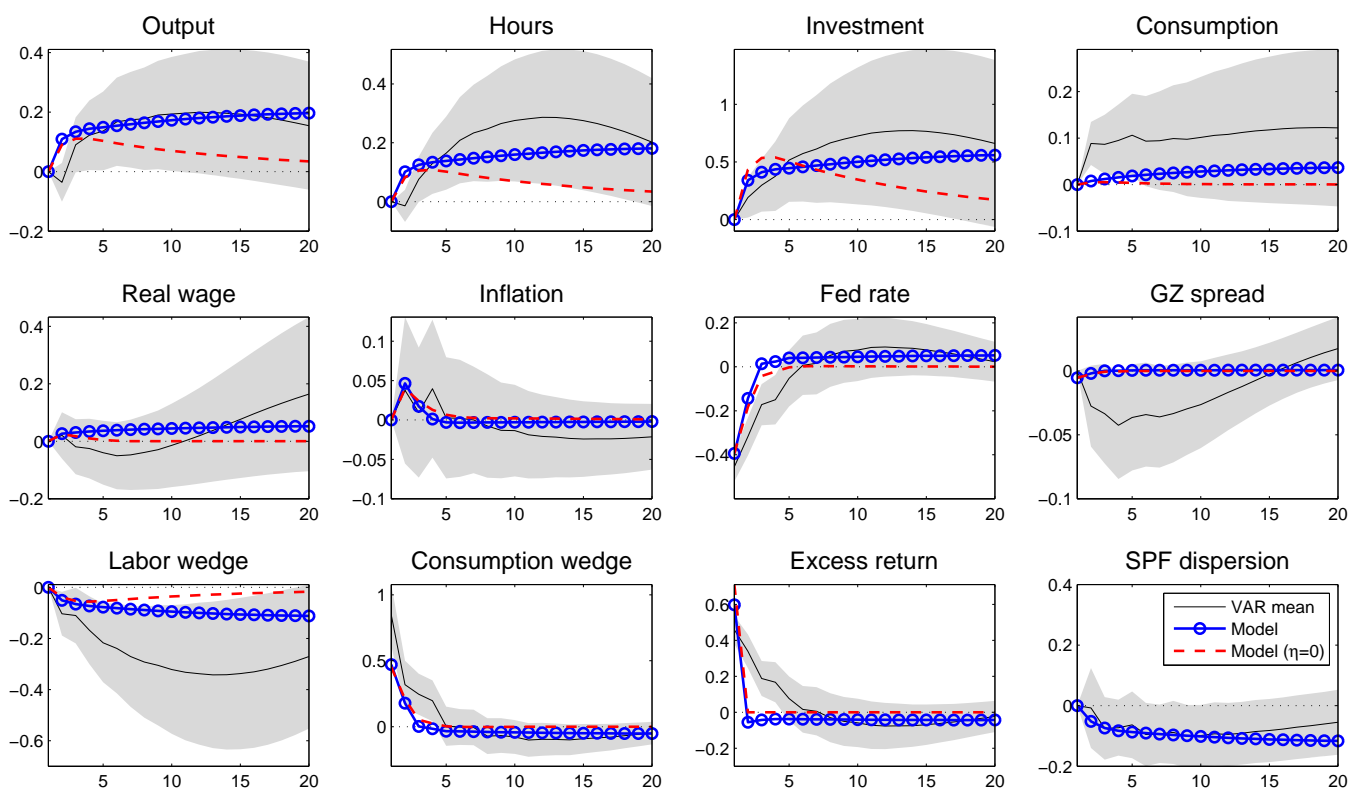


Notes: See notes from Figure 4.

The main reason for the failure of the RE model arises from the high degree of consumption habit: at the posterior mode,  $b = 0.83$ . This value is in line with estimates found in the New Keynesian literature, such as Christiano et al. (2005) and Smets and Wouters (2007). As pointed out by Christiano et al. (2005), the high value of  $b$  allows the model to accommodate the main property of an expansionary monetary policy shock (Figure 5): consumption grows while the interest rate is falling. While this negative co-movement between consumption and

interest rate helps the RE model match the responses to a monetary policy shock, it becomes inconsistent with the responses to the financial shock.<sup>41</sup> In order to strike a balance between matching consumption and interest rates, the three shock estimation chooses parameter values so that both variables remain roughly constant in response to a financial shock. In turn, this implies that an expansionary monetary policy shock raises consumption only slightly in the estimated RE model.

Figure 6: Responses to a monetary policy shock: turning off ambiguity



*Notes:* The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence bands. The blue circled lines are the impulse responses from the baseline model with ambiguity, estimated using the VAR responses to all three structural shocks (technology, financial and monetary policy). The red dashed lines are the counterfactual responses where we set the entropy constraint  $\eta$  to 0, while holding other parameters at the estimated values. The responses of output, hours, investment, consumption and real wages are in percentage deviations from the steady states while inflation, fed rate, GZ spread and excess return are in annual percentage points. The rest are in quarterly percentage points.

<sup>41</sup>Note that this tension did not exist in the single shock estimation. Matching more conditional dynamics may explain why in the medium-scale DSGE literature shocks to the return of investing are typically not found to produce co-movement (see Justiniano et al. (2011)), in contrast to matching impulse responses conditional only on the financial shock (as in Gilchrist and Zakrajšek (2011)).

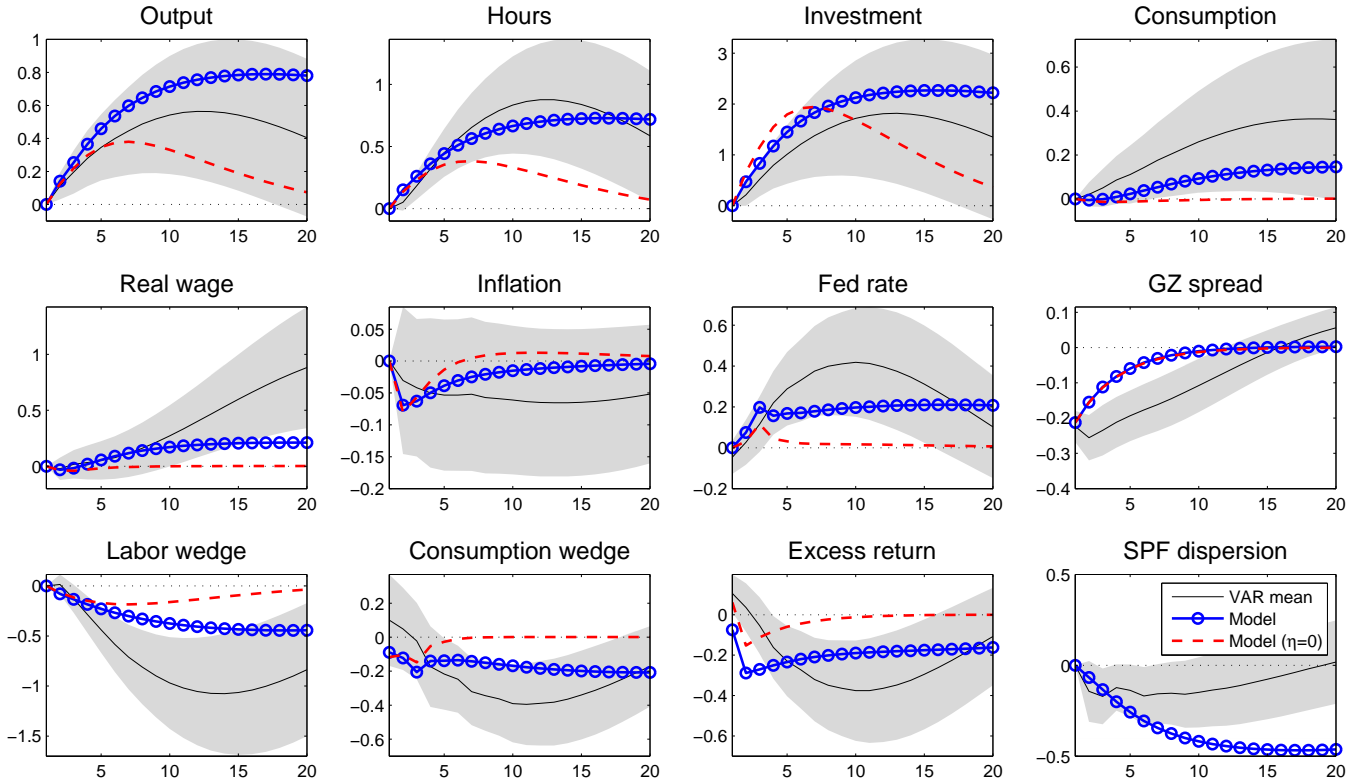
Why, then, can our model with ambiguity simultaneously match the VAR responses for a financial shock and a monetary policy shock, as shown in Figure 5? The success is due to two factors. First, as confidence accumulates, the demand for safe assets falls and hence makes it possible for high consumption and high interest rates to co-exist. This allows the model to account for the impulse responses to a financial shock as well as the medium-run dynamics for a monetary policy shock, when the real interest rate overshoots.

Second, the model relies largely on confidence to propagate a monetary policy shock. To see this, in Figure 6 we report the impulse responses to a monetary policy shock in our model along with the impulse responses when we shut down ambiguity, holding other parameters at their estimated values. When we turn off confidence, the real effect of a monetary policy shock is small and transitory. Consider now the response with ambiguity. In the short-run, the effect of consumption habit dominates and hence the fall in the interest rate is associated with a rise in consumption, manifested as a positive consumption wedge. As the initial expansion in economic activity raises confidence, the confidence channel overcomes the habit channel: consumption continues to rise as the real interest rate turns positive, which in turn shows up as a negative consumption wedge. In the medium run, this feedback loop between economic activity and uncertainty dominates the propagation of a monetary policy shock and hence leads to a sizable and persistent increase in output, consumption and other such real quantities and real wages, while at the same time replicating the fall in the labor wedge and the forecast dispersion.

Finally, because the real effect of a monetary policy shock is driven by the confidence channel, our model requires smaller frictions; at the posterior mode agents prices and wages are adjusted every 2.7 and 1.8 quarters, respectively, while in the RE model the corresponding numbers are 4.5 and 3.4 quarters, respectively. In addition, the estimated investment adjustment cost  $\kappa$  is significantly lower at 0.22 compared to  $\kappa = 0.73$  and the consumption habit is  $b = 0.78$  compared to 0.83 in the RE model.

We conclude by briefly discussing four additional results. First, we consider what happens to the impulse response to a financial shock when we turn off ambiguity, holding other parameters at their estimated values (Figure 7). Confidence amplifies and propagates the real effects of financial shocks while inducing co-movement and generating a fall in the wedges. Second, we report the responses to a technology shock in Figures 11 and 12 in the Appendix. In the VAR, a positive technology shock raises output, investment, consumption but slightly reduces hours in the short-run, in line with the conventional finding in the literature such as Galí (1999). We find that both ambiguity and the RE model fit the VAR reasonably well; in particular, the relatively moderate degree of estimated real and nominal rigidities in the ambiguity model is sufficient to generate the short-run decline in hours. In addition,

Figure 7: Responses to a financial shock: turning off ambiguity



Notes: See notes from Figure 6.

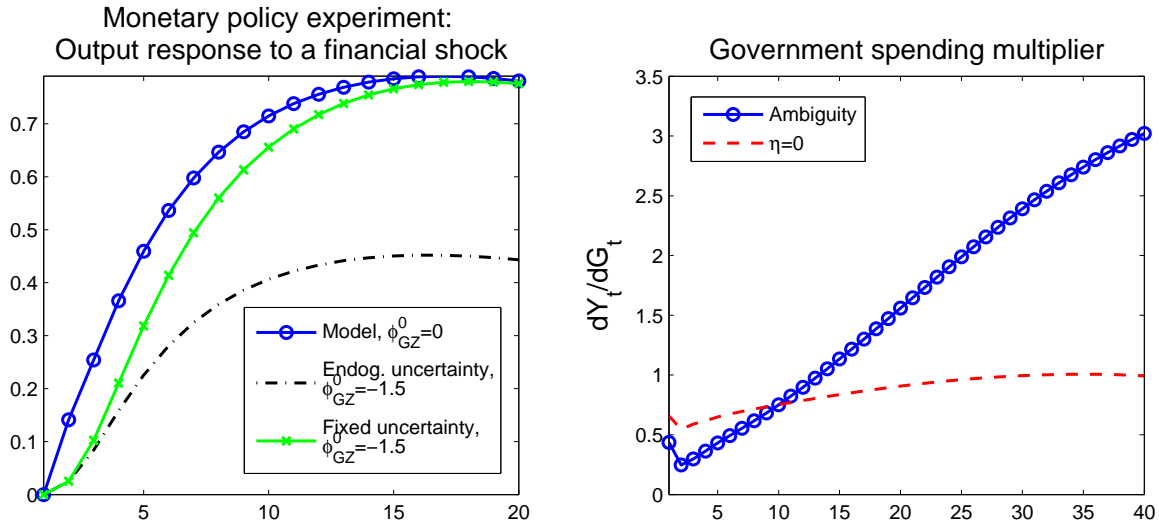
the ambiguity model can generate the fall in the dispersion of forecasts that is in line with the VAR. Third, our ambiguity model beats the RE model in terms of marginal likelihood, which penalizes more parameters, by  $(-1487 - (-1714)) = 227$  log points. Finally, we evaluate the model's prediction for utilization by augmenting the original VAR with capacity utilization series published by the Federal Reserve Board and comparing the response of utilization to a financial shock against the ambiguity model (Figure 13 in the Appendix). Although utilization is not directly targeted, the model responses line up well with the VAR for both single and three shock estimations. While utilization is quite elastic in the ambiguity model (Table 2), it is reassuring that the quantitative success of our model is not driven by a counterfactually large response of utilization.

## 4.5 Policy implications

The fact that in our model uncertainty is endogenous has important policy implications. To illustrate this, we conduct two policy experiments, using throughout the analysis parameter

values that are based on the three shock estimation. First, we evaluate the impact of modifying the Taylor rule to incorporate an adjustment to the credit spread. In the left panel of Figure 8 we report the impulse response of output to the financial shock in the ambiguity model as we keep all parameters at their baseline estimated values, but change the Taylor rule coefficient on the credit spread  $\phi_{GZ}^0$  from its original value of zero. The output effect decreases when monetary policy responds aggressively to the spread movements. For example, the peak output response of the one-standard-deviation financial shock falls by 50% from 0.8 percent to 0.4 percent when  $\phi_{GZ}^0$  decreases from zero to  $-1.5$  (black dashed line).

Figure 8: Policy experiments



*Notes:* The left panel plots the output response to a financial shock. The blue circled line is the baseline model with ambiguity, estimated using the VAR responses to all three structural shocks. The black dashed line is the counterfactual where the Taylor rule coefficient on the GZ spread is  $\phi_{GZ}^0 = -1.5$ . The green line is the response when  $\phi_{GZ}^0 = -1.5$  but with the path of uncertainty fixed at the original one. The right panel plots the government spending multiplier for output. The economy is hit by a positive spending shock at  $t = 1$  and the path of government spending follows an AR(1) process. The blue circled line is the multiplier from the baseline model with ambiguity, estimated using the VAR responses to all three structural shocks. The red dashed line is the multiplier where  $\eta = 0$ , holding other parameters at the estimated values.

Much of the reduction in the output effect comes from stabilizing the endogenous variation in uncertainty. To see this, we show the effects of policy changes in the economy where the path of uncertainty is fixed to the original one. In this economy, a change in  $\phi_{GZ}^0$  has a much smaller effect. Indeed, the peak output effect of a financial shock roughly stays around the original value of 0.8 percent even when the central bank reacts with  $\phi_{GZ}^0 = -1.5$  (green line).

Second, we consider fiscal policy effects. In standard models, an increase in government spending crowds out consumption and hence the government spending multiplier on output,  $dY_t/dG_t$ , tends to be modest and below one. In our model, however, an increase in hours worked triggered by an increase in government spending raises agents' confidence, which feeds

back and raises the level of consumption and other economic activities. Because of this amplification effect, the government spending multiplier could be larger and above one. In the right panel of Figure 8, we plot the multiplier in our estimated model after a one-time, positive shock to government spending at  $t = 1$ .<sup>42</sup> The model predicts a multiplier that becomes larger than one after three years and stays persistently and significantly above one. In contrast, in a counterfactual economy where  $\eta = 0$  and other parameters are at their estimated values, the multiplier stays persistently below or around one.<sup>43</sup>

It is important to emphasize that the large effects of government spending on output are not welfare increasing even though it arises due to a reduction in uncertainty. Indeed, since in this model learning arises at firm-level there are no information externalities that the government can correct. This is in contrast to models where learning occurs through observing the aggregate economy and it highlights the importance of modeling the underlying source of uncertainty for evaluating policies. At a more general level, the comparisons of these counterfactual models in the monetary and fiscal policy experiments underscore the importance for policy analysis of modeling time-variation in uncertainty as an endogenous response that in turn further affects economic decisions.

## 4.6 Evidence from firm-level survey data

We provide a further test of the model by comparing our model-implied firm-level confidence process with the time-series moments of uncertainty directly measured from the micro survey data. Our measure of confidence is the cross-sectional average dispersion of firm-level capital return forecasts. We use a series constructed by Senga (2015) using I/B/E/S and Compustat data.<sup>44</sup> For each firm, Senga (2015) measures the min-max range across analysts' forecasts of the return on capital for that firm. Taking the cross-sectional average across firms of that forecast range results in a time-series measure. For the model counterpart, we calculate the range of capital return forecasts by computing expected capital returns implied by the set of productivity process (2.11).<sup>45</sup>

Table 4 reports several moments from the data and the model: the correlation between the forecast range and real GDP, the time-series fluctuation of the range measured by its standard deviation, and the ratio of the standard deviation of range to that of real GDP.

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<sup>42</sup>We assume that the government spending  $G_t$  in the resource constraint (2.10) is given by  $G_t = g_t Y_t$ , where  $g_t$  follows  $\ln g_t = (1 - 0.95) \ln \bar{g} + 0.95 \ln g_{t-1} + \epsilon_{g,t}$ .

<sup>43</sup>We also computed a multiplier in the re-estimated RE model, where we set  $\eta = 0$  and re-estimated the remaining parameters, and found that there the multiplier also stays below or close to one.

<sup>44</sup>We thank Tatsuro Senga for generously sharing his data.

<sup>45</sup>In terms of mapping the model to data, the idea here is that the representative agent samples experts' forecast when making decisions. Stronger disagreement among experts about conditional firm-level mean returns is reflected in the agent's lower confidence in probability assessments.



Table 4: Forecast dispersion at the firm-level

	Corr(range,rgdp)	Std. range	(Std. range)/(Std. rgdp)
Data	-0.49	15.2	3.5
Model	-0.98	11.5	2.0

*Notes:* The second column reports the correlation between forecast range and real GDP. The third column reports the standard deviation of forecast range, which is logged and multiplied by 100 so that it is expressed in percentage terms. The fourth column reports the ratio of the standard deviation of range to the standard deviation of real GDP. All variables are linearly de-trended. The model moments are obtained by simulating the model and are annualized so that it matches the frequency of the data by Senga (2015).

The model can account for roughly  $(11.5/15.2=)$  75% of the time-series variation in the forecast dispersion. As in the data, the dispersion and real GDP is negatively correlated, although the model overstates the negative correlation. On the one hand, in the model uncertainty is driven solely by changes in economic activity, thus producing a strong negative co-movement. On the other hand, the firm-level data could be subject to measurement errors, which tend to bias the correlation between the forecast range and real GDP towards zero, while increasing the measured standard deviation of the range. To conclude, the time-series properties of uncertainty implied from our estimated model broadly matches firm-level data that was not used in the estimation. This external validation provides additional evidence that the estimated endogenous uncertainty mechanism is empirically plausible.

## 5 Conclusion

In this paper, we build a tractable heterogeneous-firm business cycle model where firms face Knightian uncertainty about their profitability and learn it through production. We show how, even in the absence of any other frictions, the feedback mechanism endogenously generates empirically desirable cross-equation restrictions such as: co-movement driven by demand shocks, amplified and hump-shaped dynamics, and countercyclical correlated wedges in the equilibrium conditions for labor, risk-free and risky assets. We embed our learning mechanism into a standard medium-scale model and estimate it by matching impulse responses of macroeconomic aggregates and asset prices to financial, monetary policy and technology shocks. We find that our model improves on conventional models in replicating impulse responses, requires less real and nominal rigidities and predicts magnified responses of economic activity to monetary and fiscal policies, while at the same time producing a confidence process that is consistent with the survey data both at the macro and micro level.

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## 6 Appendix (For online publication)

### 6.1 Recursive competitive equilibrium for the frictionless model

We collect exogenous aggregate state variables (such as economy-wide TFP) in a vector  $X$  with a cumulative transition function  $F(X'|X)$ . The endogenous aggregate state is the distribution of firm-level variables. A firm’s type is identified by the posterior mean estimate of productivity  $\tilde{z}_l$  and the posterior variance  $\Sigma_l$ . The worst-case TFP is not included because it is implied by the posterior mean and variance. We denote the cross-sectional distribution of firms’ type by  $\xi_1$  and  $\xi_2$ .  $\xi_1$  is a stage 1 distribution over  $(\tilde{z}_l, \Sigma_l)$  and  $\xi_2$  is a stage 2 distribution over  $(\tilde{z}'_l, \Sigma'_l)$ .  $\xi'_1$ , in turn, is a distribution over  $(\tilde{z}'_l, \Sigma'_l)$  at stage 1 in the next period.<sup>46</sup>

First, consider the household’s problem. The household’s wealth can be summarized by a portfolio  $\vec{\theta}_l$  which consists of share  $\theta_l$  for each firm, capital stock  $K$  and the riskless bond holdings  $B$ . We use  $V_1^h$  and  $V_2^h$  to denote the household’s value function at stage 1 and stage 2, respectively. We use  $m$  to summarize the income available to the household at stage 2. The household’s problem at stage 1 is

$$V_1^h(\vec{\theta}_l, K, B; \xi_1, X) = \max_H \left\{ -\frac{H^{1+\phi}}{1+\phi} + E^*[V_2^h(\hat{m}; \hat{\xi}_2, X)] \right\} \quad (6.1)$$

$$\text{s.t. } \hat{m} = WH + r^K K + RB + \int (\hat{D}_l + \hat{P}_l)\theta_l dl$$

where we momentarily use the *hat* symbol to indicate random variables that will be resolved

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<sup>46</sup>See also Senga (2015) for a recursive representation of an imperfect information heterogeneous-firm model with time-varying uncertainty.

at stage 2. The household's problem at stage 2 is

$$\begin{aligned}
V_2^h(m; \xi_2, X) &= \max_{C, \vec{\theta}_l', K', B'} \left\{ \ln C + \beta \int V_1^h(\vec{\theta}_l', K', B'; \xi_1', X') dF(X'|X) \right\} \\
\text{s.t. } & C + K' - (1 - \delta)K + B' + \int P_l \theta_l' dl \leq m \\
& \xi_1' = \Gamma(\xi_2, X)
\end{aligned} \tag{6.2}$$

In problem (6.1), households choose labor supply based on the worst-case stage 2 value (recall that we use  $E^*$  to denote worst-case conditional expectations). The problem (6.2), in turn, describes the household's consumption and asset allocation problem given the realization of income and aggregate states. In particular, they take as given the law of motion of the next period's distribution  $\xi_1' = \Gamma(\xi_2, X)$ , which in equilibrium is consistent with the firm's policy function. Importantly, in contrast to the stage 2 problem, a law of motion that describes the evolution of  $\xi_2$  from  $(\xi_1, X)$  is absent in the stage 1 problem. Indeed, if there is no ambiguity in the model, agents take as given the law of motion  $\xi_2 = \Upsilon(\xi_1, X)$ , which in equilibrium is consistent with the firm's policy function and the true data generating process of the firm-level profitability. Since agents are ambiguous about each firm's profitability process, they cannot settle on a single law of motion about the distribution of firms. Finally, the continuation value at stage 2 is governed by the transition density of aggregate exogenous states  $X$ .

Next, consider the firms' problem. We use  $v_1^f$  and  $v_2^f$  to denote the firm's value function at stage 1 and stage 2, respectively. Firm  $l$ 's problem at stage 1 is

$$\begin{aligned}
v_1^f(\tilde{z}_l, \Sigma_l; \xi_1, X) &= \max_{H_l, K_l} E^*[v_2^f(\tilde{z}_l', \Sigma_l'; \hat{\xi}_2, X)] \\
\text{s.t. } & \text{Updating rules (2.7) and (2.8)}
\end{aligned} \tag{6.3}$$

and firm  $l$ 's value at stage 2 is

$$\begin{aligned}
v_2^f(\tilde{z}_l', \Sigma_l'; \xi_2, X) &= \lambda(Y^{\frac{1}{\theta}} Y_l^{1-\frac{1}{\theta}} - WH_l - r^K K_l) + \beta \int v_1^f(\tilde{z}_l', \Sigma_l'; \xi_1', X') dF(X'|X) \\
\text{s.t. } & \xi_1' = \Gamma(\xi_2, X)
\end{aligned} \tag{6.4}$$

where we simplify the exposition by expressing a firm's value in terms of the marginal utility  $\lambda$  of the representative household. Similar to the household's problem, a firm's problem at stage 1 is to choose the labor and capital demand so as to maximize the worst-case stage 2 value. Note that the posterior mean  $\tilde{z}_l'$  will be determined by the realization of output  $Y_l$  at stage 2 while the posterior variance  $\Sigma_l'$  is determined by  $\Sigma_l$  and the input level at stage 1.

The recursive competitive equilibrium is therefore a collection of value functions, policy

functions, and prices such that

1. Households and firms optimize; (6.1) – (6.4).
2. The labor market, goods market, and asset markets clear.
3. The law of motion  $\xi_1' = \Gamma(\xi_2, X)$  is induced by the firms' policy functions.

## 6.2 Solution procedure

Here we describe the general solution procedure of the model. First, we derive the law of motion assuming that the model is a rational expectations model where the worst case expectations are on average correct. Second, we take the equilibrium law of motion formed under ambiguity and then evaluate the dynamics under the econometrician's data generating process. We provide a step-by-step description of the procedure:

1. Find the worst-case steady state.

We first compute the steady state of the filtering problem (2.7), (2.8), and (2.11), under the worst-case mean to find the firm-level TFP at the worst-case steady state,  $\bar{z}^0$ . We then solve the steady state for other equilibrium conditions evaluated at  $\bar{z}^0$ .

2. Log-linearize the model around the worst-case steady state.

We can solve for the dynamics using standard tools for linear rational expectation models. We base our discussion based on the method proposed by Sims (2002).

We first need to deal with the issue that idiosyncratic shocks realize at the beginning of stage 2. Handling this issue correctly is important, since variables chosen at stage 1, such as input choice, should be based on the worst-case TFP, while variables chosen at stage 2, such as consumption and investment, would be based on the realized TFP (but also on the worst-case future TFP). To do this, we exploit the certainty equivalence property of linear decision rules. We first solve for decision rules *as if* both aggregate and idiosyncratic shocks realize at the beginning of the period. We call them “pre-production decision rules”. We then solve for decision rules *as if* (i) both aggregate and idiosyncratic shocks realize at the beginning of the period and (ii) stage 1 variables are pre-determined. We call them “post-production decision rules”. Finally, when we characterize the dynamics from the perspective of the econometrician, we combine the pre-production and post-production decision rules and obtain an equilibrium law of motion.



To obtain pre-production decision rules, we collect the linearized equilibrium conditions, which include firm-level conditions, into the canonical form:

$$\mathbf{\Gamma}_0^{pre} \hat{\mathbf{y}}_t^{pre,0} = \mathbf{\Gamma}_1^{pre} \hat{\mathbf{y}}_{t-1}^{pre,0} + \mathbf{\Psi}^{pre} \omega_t + \mathbf{\Upsilon}^{pre} \eta_t^{pre},$$

where  $\hat{\mathbf{y}}_t^{pre,0}$  is a column vector of size  $k$  that contains all variables and the conditional expectations.  $\hat{\mathbf{y}}_t^{pre,0} = \mathbf{y}_t^{pre} - \bar{\mathbf{y}}^0$  denotes deviations from the worst-case steady state and  $\eta_t$  are expectation errors, which we define as  $\eta_t^{pre} = \hat{\mathbf{y}}_t^{pre,0} - E_{t-1}^* \hat{\mathbf{y}}_t^{pre,0}$  such that  $E_{t-1}^* \eta_t^{pre} = 0$ . We define  $\omega_t = [e_{l,t} \ e_t]'$ , where  $e_{l,t} = [\epsilon_{z,l,t} \ u_{l,t} \ \nu_{l,t}]'$  is a vector of idiosyncratic shocks and  $e_t$  is a vector of aggregate shocks of size  $n$ .

The vector  $\hat{\mathbf{y}}_t^{pre,0}$  contains firm-level variables such as firm  $l$ 's labor input,  $H_{l,t}$ . In contrast to other linear heterogeneous-agent models with imperfect information such as Lorenzoni (2009), all agents share the same information set. Thus, to derive the aggregate law of motion, we simply aggregate over firm  $l$ 's linearized conditions and replace firm-specific variables with their cross-sectional means (e.g., we replace  $H_{l,t}$  with  $H_t \equiv \int_0^1 H_{l,t} dl$ ) and set  $e_{l,t} = \mathbf{0}$ , which uses the law of large numbers for idiosyncratic shocks.

We order variables in  $\hat{\mathbf{y}}_t^{pre,0}$  as

$$\hat{\mathbf{y}}_t^{pre,0} = \begin{bmatrix} \hat{\mathbf{y}}_{1,t}^{pre,0} \\ \hat{\mathbf{y}}_{2,t}^{pre,0} \\ \hat{\mathbf{s}}_t^{pre,0} \end{bmatrix},$$

where  $\hat{\mathbf{y}}_{1,t}^{pre,0}$  is a column vector of size  $k_1$  of variables determined at stage 1,  $\hat{\mathbf{y}}_{2,t}^{pre,0}$  is a column vector of size  $k_2$  of variables determined at stage 2, and  $\hat{\mathbf{s}}_t^{pre,0} = [\hat{s}_{1,t}^{pre,0} \ \hat{s}_{2,t}^{pre,0}]'$ , where  $s_{1,t} = \bar{z} - E_{t-1}^* z_t$  and  $s_{2,t} = \bar{z} - \tilde{z}_{t|t}$ .

The resulting solution of pre-production decision rules is obtained applying the method developed by Sims (2002):

$$\hat{\mathbf{y}}_t^{pre,0} = \mathbf{T}^{pre} \hat{\mathbf{y}}_{t-1}^{pre,0} + \mathbf{R}^{pre} [\mathbf{0}_{3 \times 1} \ e_t]', \quad (6.5)$$

where  $\mathbf{T}^{pre}$  and  $\mathbf{R}^{pre}$  are  $k \times k$  and  $k \times (n+3)$  matrices, respectively.

The solution of post-production decision rules can be obtained in a similar way by first collecting the equilibrium conditions into the canonical form

$$\mathbf{\Gamma}_0^{post} \hat{\mathbf{y}}_t^{post,0} = \mathbf{\Gamma}_1^{post} \hat{\mathbf{y}}_{t-1}^{post,0} + \mathbf{\Psi}^{post} \omega_t + \mathbf{\Upsilon}^{post} \eta_t^{post},$$

and is given by

$$\hat{\mathbf{y}}_t^{post,0} = \mathbf{T}^{post} \hat{\mathbf{y}}_{t-1}^{post,0} + \mathbf{R}^{post} [\mathbf{0}_{3 \times 1} \quad e_t]', \quad (6.6)$$

where

$$\hat{\mathbf{y}}_t^{post,0} = \begin{bmatrix} \hat{\mathbf{y}}_{1,t}^{post,0} \\ \hat{\mathbf{y}}_{2,t}^{post,0} \\ \hat{\mathbf{s}}_t^{post,0} \end{bmatrix},$$

and  $\mathbf{T}^{post}$  and  $\mathbf{R}^{post}$  are  $k \times k$  and  $k \times (n+3)$  matrices, respectively.

### 3. Characterize the dynamics from the econometrician's perspective.

The above law of motion was based on the worst-case probabilities. We need to derive the equilibrium dynamics under the true DGP, where the cross-sectional mean of firm-level TFP is  $\bar{z}$ . We are interested in two objects: the zero-risk steady state and the dynamics around that zero-risk steady state.

#### (a) Find the zero-risk steady state.

This the fixed point  $\bar{\mathbf{y}}$  where the decision rules (6.5) and (6.6) are evaluated at the realized cross-sectional mean of firm-level TFP  $\bar{z}$ :

$$\begin{aligned} \bar{\mathbf{y}}^{pre} - \bar{\mathbf{y}}^0 &= \mathbf{T}^{pre} (\bar{\mathbf{y}} - \bar{\mathbf{y}}^0), \\ \bar{\mathbf{y}}^{post} - \bar{\mathbf{y}}^0 &= \mathbf{T}^{post} (\bar{\mathbf{y}} - \bar{\mathbf{y}}^0) + \mathbf{R}^{post} [\bar{\mathbf{s}} \quad \mathbf{0}_{(n+1) \times 1}]', \end{aligned} \quad (6.7)$$

where

$$\bar{\mathbf{y}} = \begin{bmatrix} \bar{\mathbf{y}}_1^{pre} \\ \bar{\mathbf{y}}_2^{post} \\ \bar{\mathbf{s}}^{post} \end{bmatrix}.$$

Note that we do not feed in the realized firm-level TFP to the pre-production decision rules since idiosyncratic shocks realize at the beginning of stage 2.

We obtain  $\bar{\mathbf{s}}$  from

$$\bar{\mathbf{s}} = [\mathbf{T}_{3,1}^{post} \quad \mathbf{T}_{3,2}^{post} \quad \mathbf{T}_{3,3}^{post}] (\bar{\mathbf{y}} - \bar{\mathbf{y}}^0) + \bar{\mathbf{s}}^0,$$

where

$$\mathbf{T}^{post} = \begin{bmatrix} \mathbf{T}_{1,1}^{post} & \mathbf{T}_{1,2}^{post} & \mathbf{T}_{1,3}^{post} \\ (k_1 \times k_1) & (k_1 \times k_2) & (k_1 \times 2) \\ \mathbf{T}_{2,1}^{post} & \mathbf{T}_{2,2}^{post} & \mathbf{T}_{2,3}^{post} \\ (k_2 \times k_1) & (k_2 \times k_2) & (k_2 \times 2) \\ \mathbf{T}_{3,1}^{post} & \mathbf{T}_{3,2}^{post} & \mathbf{T}_{3,3}^{post} \\ (2 \times k_1) & (2 \times k_2) & (2 \times 2) \end{bmatrix}.$$

(b) Dynamics around the zero-risk steady state.

Denoting  $\hat{\mathbf{y}}_t \equiv \mathbf{y}_t - \bar{\mathbf{y}}$  the deviations from the zero-risk steady state, we combine the decision rules (6.5) and (6.6) evaluated at the true DGP and the equations for the zero-risk steady state (6.7):

$$\hat{\mathbf{y}}_t^{pre} = \mathbf{T}^{pre} \hat{\mathbf{y}}_{t-1} + \mathbf{R}^{pre} [\mathbf{0}_{3 \times 1} \quad e_t]', \quad (6.8)$$

$$\hat{\mathbf{y}}_t^{post} = \mathbf{T}^{post} [\hat{\mathbf{y}}_{1,t}^{pre} \quad \hat{\mathbf{y}}_{2,t-1} \quad \hat{\mathbf{s}}_{t-1}]' + \mathbf{R}^{post} [\hat{\mathbf{s}}_t \quad 0 \quad e_t]', \quad (6.9)$$

$$\hat{\mathbf{s}}_t = [\mathbf{T}_{3,1}^{post} \quad \mathbf{T}_{3,2}^{post} \quad \mathbf{T}_{3,3}^{post}] [\hat{\mathbf{y}}_{1,t}^{pre} \quad \hat{\mathbf{y}}_{2,t-1} \quad \hat{\mathbf{s}}_{t-1}]' + \mathbf{R}_{3,3}^{post} [\mathbf{0}_{3 \times 1} \quad e_t]', \quad (6.10)$$

and

$$\hat{\mathbf{y}}_t = \begin{bmatrix} \hat{\mathbf{y}}_{1,t}^{pre} \\ \hat{\mathbf{y}}_{2,t}^{post} \\ \hat{\mathbf{s}}_t^{post} \end{bmatrix}, \quad (6.11)$$

where

$$\mathbf{R}^{post} = \begin{bmatrix} \mathbf{R}_{1,1}^{post} & \mathbf{R}_{1,2}^{post} & \mathbf{R}_{1,3}^{post} \\ (k_1 \times 2) & (k_1 \times 1) & (k_1 \times n) \\ \mathbf{R}_{2,1}^{post} & \mathbf{R}_{2,2}^{post} & \mathbf{R}_{2,3}^{post} \\ (k_2 \times 2) & (k_2 \times 1) & (k_2 \times n) \\ \mathbf{R}_{3,1}^{post} & \mathbf{R}_{3,2}^{post} & \mathbf{R}_{3,3}^{post} \\ (2 \times 2) & (2 \times 1) & (2 \times n) \end{bmatrix}.$$

We combine equations (6.8), (6.9), (6.10), and (6.11) to obtain the equilibrium law of motion. To do so, we first define submatrices of  $\mathbf{T}^{pre}$  and  $\mathbf{R}^{pre}$ :

$$\mathbf{T}^{pre} = \begin{bmatrix} \mathbf{T}_1^{pre} \\ (k_1 \times k) \\ \mathbf{T}_2^{pre} \\ (k_2 \times k) \\ \mathbf{T}_3^{pre} \\ (2 \times k) \end{bmatrix}, \quad \mathbf{R}^{pre} = \begin{bmatrix} \mathbf{R}_{1,1}^{pre} & \mathbf{R}_{1,2}^{pre} \\ (k_1 \times 3) & (k_1 \times n) \\ \mathbf{R}_{2,1}^{pre} & \mathbf{R}_{2,2}^{pre} \\ (k_2 \times 3) & (k_2 \times n) \\ \mathbf{R}_{3,1}^{pre} & \mathbf{R}_{3,2}^{pre} \\ (2 \times 3) & (2 \times n) \end{bmatrix}.$$

A  $k \times k$  matrix  $\mathbf{T}$  is then given by

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_1^{pre} \\ \mathbf{T}_2 \\ \mathbf{T}_3 \end{bmatrix},$$

where  $\mathbf{T}_2$  and  $\mathbf{T}_3$  are given by

$$\begin{aligned}\mathbf{T}_2 &= [\mathbf{Q}_{2,1} \quad \mathbf{Q}_{2,2} + \mathbf{T}_{2,2}^{post} + \mathbf{R}_{2,1}^{post} \mathbf{T}_{3,2}^{post} \quad \mathbf{Q}_{2,3} + \mathbf{T}_{2,3}^{post} + \mathbf{R}_{2,1}^{post} \mathbf{T}_{3,3}^{post}], \\ \mathbf{T}_3 &= [\mathbf{Q}_{3,1} \quad \mathbf{Q}_{3,2} + \mathbf{T}_{3,2}^{post} + \mathbf{R}_{3,1}^{post} \mathbf{T}_{3,2}^{post} \quad \mathbf{Q}_{3,3} + \mathbf{T}_{3,3}^{post} + \mathbf{R}_{3,1}^{post} \mathbf{T}_{3,3}^{post}],\end{aligned}$$

and  $\mathbf{Q}_{2,1}$ ,  $\mathbf{Q}_{2,2}$ , and  $\mathbf{Q}_{2,3}$  are  $k_2 \times k_1$ ,  $k_2 \times k_2$ , and  $k_2 \times 2$  submatrices of  $\mathbf{Q}_2$ , where  $\mathbf{Q}_2 \equiv (\mathbf{T}_{2,1}^{post} + \mathbf{R}_{2,1}^{post} \mathbf{T}_{3,1}^{post}) \mathbf{T}_1^{pre}$ , so that  $\mathbf{Q}_2 = [\mathbf{Q}_{2,1} \quad \mathbf{Q}_{2,2} \quad \mathbf{Q}_{2,3}]$ . Similarly,  $\mathbf{Q}_{3,1}$ ,  $\mathbf{Q}_{3,2}$ , and  $\mathbf{Q}_{3,3}$  are  $k_3 \times k_1$ ,  $k_3 \times k_2$ , and  $k_3 \times 2$  submatrices of  $\mathbf{Q}_3$ , where  $\mathbf{Q}_3 \equiv (\mathbf{T}_{3,1}^{post} + \mathbf{R}_{3,1}^{post} \mathbf{T}_{3,1}^{post}) \mathbf{T}_1^{pre}$ , so that  $\mathbf{Q}_3 = [\mathbf{Q}_{3,1} \quad \mathbf{Q}_{3,2} \quad \mathbf{Q}_{3,3}]$ .

A  $k \times n$  matrix  $\mathbf{R}$  is given by

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{1,2}^{pre} \\ \mathbf{R}_2 \\ \mathbf{R}_3 \end{bmatrix},$$

where

$$\begin{aligned}\mathbf{R}_2 &= \mathbf{T}_{2,1}^{post} \mathbf{R}_{1,2}^{pre} + \mathbf{R}_{2,1}^{post} (\mathbf{T}_{3,1}^{post} \mathbf{R}_{1,2}^{pre} + \mathbf{R}_{3,3}^{post}) + \mathbf{R}_{2,3}^{post}, \\ \mathbf{R}_3 &= \mathbf{T}_{3,1}^{post} \mathbf{R}_{1,2}^{pre} + \mathbf{R}_{3,1}^{post} (\mathbf{T}_{3,1}^{post} \mathbf{R}_{1,2}^{pre} + \mathbf{R}_{3,3}^{post}) + \mathbf{R}_{3,3}^{post}.\end{aligned}$$

The equilibrium law of motion is then given by

$$\hat{\mathbf{y}}_t = \mathbf{T} \hat{\mathbf{y}}_{t-1} + \mathbf{R} e_t.$$

### 6.3 Illustration of log-linearization and effects of idiosyncratic uncertainty

In what follows we explain the log-linearizing logic by simple expressions for the expected worst-case output at stage 1 (pre-production) and the realized output at stage 2 (post-production). We use the example to illustrate that uncertainty about the firm-level productivity has a first-order effect at the aggregate level. To do so, we first log-linearize the expected worst-case output of firm  $l$  at stage 1, as described in section Appendix 6.2

$$E_t^* \hat{Y}_{l,t}^0 = \hat{A}_t^0 + E_t^* \hat{z}_{l,t}^0 + \hat{F}_{l,t}^0, \quad (6.12)$$

and the realized output of individual firm  $l$  at stage 2:

$$\hat{Y}_{l,t}^0 = \hat{A}_t^0 + \hat{z}_{l,t}^0 + \hat{F}_{l,t}^0, \quad (6.13)$$

where we use  $\hat{x}_t^0 = x_t - \bar{x}^0$  to denote log-deviations from the worst-case steady state and set the trend growth rate  $\gamma$  to zero to ease notation. The worst-case individual output (6.12) is the sum of three components: the current level of economy-wide TFP, the worst-case individual TFP, and the input level. The realized individual output (6.13), in turn, is the sum of economy-wide TFP, the *realized* individual TFP, and the input level.

We then aggregate the log-linearized individual conditions (6.12) and (6.13) to obtain the cross-sectional mean of worst-case individual output:

$$E_t^* \hat{Y}_t^0 = \hat{A}_t^0 + E_t^* \hat{z}_t^0 + \hat{F}_t^0, \quad (6.14)$$

and the cross-sectional mean of realized individual output:

$$\hat{Y}_t^0 = \hat{A}_t^0 + \hat{z}_t^0 + \hat{F}_t^0, \quad (6.15)$$

where we simply eliminate subscript  $l$  to denote the cross-sectional mean, i.e.,  $\hat{x}_t^0 \equiv \int_0^1 \hat{x}_{l,t}^0 dl$ .

We now characterize the dynamics under the true DGP. To do this, we feed in the cross-sectional mean of individual TFP, which is constant under the true DGP, into (6.14) and (6.15). Using (6.14), the cross-sectional mean of worst-case output is given by

$$E_t^* \hat{Y}_t = \hat{A}_t + E_t^* \hat{z}_t + \hat{F}_t, \quad (6.16)$$

where we use  $\hat{x}_t = x_t - \bar{x}$  to denote log-deviations from the steady-state under the true DGP. Using (6.15), the realized aggregate output is given by

$$\hat{Y}_t = \hat{A}_t + \hat{F}_t, \quad (6.17)$$

where we used  $\hat{z}_t = 0$  under the true DGP. Importantly,  $E_t^* \hat{z}_t$  in (6.17) is not necessarily zero outside the steady state. To see this, combine (2.11) and (2.15) and log-linearize to obtain an expression for  $E_t^* \hat{z}_{l,t}$ :

$$E_t^* \hat{z}_{l,t} = \varepsilon_{z,z} \hat{z}_{l,t-1|t-1} - \varepsilon_{z,\Sigma} \hat{\Sigma}_{l,t-1|t-1}. \quad (6.18)$$

From (2.8), the posterior variance is negatively related to the level of input  $F$ :

$$\hat{\Sigma}_{l,t-1|t-1} = \varepsilon_{\Sigma,\Sigma} \hat{\Sigma}_{l,t-2|t-2} - \varepsilon_{\Sigma,Y} \hat{F}_{l,t-1}, \quad (6.19)$$

The elasticities  $\varepsilon_{z,z}$ ,  $\varepsilon_{z,\Sigma}$ ,  $\varepsilon_{\Sigma,\Sigma}$ , and  $\varepsilon_{\Sigma,Y}$  are functions of structural parameters and are all positive. We combine (6.18) and (6.19) to obtain

$$E_t^* \hat{z}_{l,t} = \varepsilon_{z,z} \hat{z}_{l,t-1|t-1} - \varepsilon_{z,\Sigma} \varepsilon_{\Sigma,\Sigma} \hat{\Sigma}_{l,t-2|t-2} + \varepsilon_{z,\Sigma} \varepsilon_{\Sigma,Y} \hat{F}_{l,t-1}. \quad (6.20)$$

Finally, we aggregate (6.20) across all firms:

$$E_t^* \hat{z}_t = -\varepsilon_{z,\Sigma} \varepsilon_{\Sigma,\Sigma} \hat{\Sigma}_{t-2|t-2} + \varepsilon_{z,\Sigma} \varepsilon_{\Sigma,Y} \hat{F}_{t-1}, \quad (6.21)$$

where we used  $\int_0^1 \hat{z}_{l,t-1|t-1} dl = 0$ .<sup>47</sup>

Notice again that the worst-case conditional cross-sectional mean simply aggregates linearly the worst-case conditional mean,  $-a_{l,t}$ , of each firm. Since the firm-specific worst-case means are a function of idiosyncratic uncertainty, which in turn depend on the firms' scale, equation (6.21) shows that the average level of economic activity,  $\hat{F}_{t-1}$ , has a first-order effect on the cross-sectional average of the worst-case mean.

## 6.4 A stylized business cycle example

We consider a stylized model without capital to illustrate the qualitative features implied by the feedback between uncertainty and economic activity. In this simple model we make two key assumptions: (1) labor is chosen before productivity is known and (2) there is a negative relationship between current uncertainty and past labor choice.

The representative agent has the following per-period utility function

$$U(C_t, H_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \beta \frac{H_t^{1+\phi}}{1+\phi}.$$

which here extends (2.14) by allowing for a more general coefficient of relative risk aversion, and  $\phi$  is the inverse of the Frisch labor elasticity. We simplify algebra below by multiplying the disutility of labor by the discount factor  $\beta$ .

Output is produced according to  $Y_t = Z_t H_{t-1}$ . The subscript on hours reflects the assumption that labor input is chosen before the realization of productivity  $Z_t$ , which is random. The resource constraint is given by  $C_t + G_t = Y_t$ , where government spending follows an AR(1) process

$$\ln G_{t+1} = (1 - \rho) \ln \bar{G} + \rho \ln G_t + u_{g,t+1}, \quad (6.22)$$

where  $u_{g,t+1}$  is distributed  $i.i.d. N(0, \sigma_g^2)$ . We use upper bars to denote the steady states. Hence,  $\bar{G}$  is the steady-state level of government spending.

The productivity process takes the form

$$\ln Z_{t+1} = \mu_t^* + u_{z,t+1}, \quad (6.23)$$

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<sup>47</sup>This follows from aggregating the log-linearized version of (2.7) and evaluating the equation under the true DGP. Intuitively, since the cross-sectional mean of idiosyncratic TFP is constant, the cross-sectional mean of the Kalman posterior mean estimate is a constant as well.

where  $u_z$  is an iid sequence of shocks, normally distributed with mean zero and variance  $\sigma_z^2$ . The sequence  $\mu$  is deterministic and unknown to agents (see Ilut and Schneider (2014) for details). The agent perceives the unknown component  $\mu_t$  to be ambiguous. We parametrize the one-step-ahead set of beliefs at date  $t$  by a set of means  $\mu_t \in [-a_t, a_t]$ . Here  $a_t$  captures agent's lack of confidence in his probability assessment of  $Z_{t+1}$ . We allow confidence itself to change over time, and in particular, we assume that  $a_t$  is negatively related to past labor:

$$a_t = \bar{a} - \zeta \hat{H}_{t-1}, \quad \zeta > 0, \quad (6.24)$$

where hats denote log-deviations from the steady states (and hence  $\hat{H}_{t-1} = \ln H_{t-1} - \ln \bar{H}$ ).

We now solve the social planner's problem, for which the Bellman equation is

$$V(H_{-1}, Z, G) = \max_H \left[ U(C, H) + \beta \min_{\mu \in [-a, a]} E^\mu V(H, Z', G') \right],$$

where the constraints are given by the production function and resource constraint. The conditional distribution of  $Z'$  under belief  $\mu$  is given by (6.23), where ambiguity evolves according to the law of motion (6.24). The transition law of the  $G$  is given by (6.22).

The worst-case belief can be easily solved for at the equilibrium consumption plan: the worst case expected productivity is low. It follows that the social planner's problem is solved under the worst case belief  $\mu = -a$ . Denoting conditional moments under the worst case belief by stars we obtain

$$H^\phi = E^* [C'^{-\sigma} Z']. \quad (6.25)$$

The optimality conditions equates the current marginal disutility of working with its expected benefit, formed under the worst-case belief. The latter is given by the marginal product of labor weighted by the marginal utility of consumption. In this stylized model we further assume that the agent does not internalize the effect of hours on the evolution of confidence.

We take logs of the optimality condition in (6.25) and substitute the log-linearized production function and resource constraint. The log-linearized decision rule of hours around the steady state relates current hours worked with the worst-case exogenous variables as

$$\hat{H}_t = \varepsilon_Z (-\hat{a}_t) + \varepsilon_G \rho \hat{G}_t.$$

Using the method of undetermined coefficients we find the elasticities  $\varepsilon_Z$  and  $\varepsilon_G$  equal to  $(1 - \sigma \lambda_Y) / (\phi + \sigma \lambda_Y)$  and  $\sigma \lambda_G / (\phi + \sigma \lambda_Y)$ , respectively, where  $\lambda_Y \equiv \bar{Y} / \bar{C}$  and  $\lambda_G \equiv \bar{G} / \bar{C}$ .

The response of optimal hours to news about expected productivity is affected by the intertemporal elasticity of consumption (IES), which here also equals the inverse of CRRA. When the IES is large enough, so that  $\sigma^{-1} > \lambda_Y$  and thus  $\varepsilon_Z > 0$ , an increase in expected

productivity raises hours. In that case the intertemporal substitution effects dominates the wealth effect that would lower hours through the effect on marginal utility.

Since expected productivity is formed under the worst-case conditional mean, and the latter is a function of past hours as in (6.24), we have

$$\hat{H}_t = \varepsilon_Z \zeta \hat{H}_{t-1} + \varepsilon_G \rho \hat{G}_t \quad (6.26)$$

Substituting the laws of motion for  $\hat{G}_t$  together with rewriting optimal hours in (6.26) for period  $t - 1$ , we have

$$\hat{H}_t = (\varepsilon_Z \zeta + \rho) \hat{H}_{t-1} - \varepsilon_Z \zeta \rho \hat{H}_{t-2} + \varepsilon_G \rho u_{g,t}. \quad (6.27)$$

Equilibrium output and consumption follow immediately as

$$\hat{Y}_t = \hat{Z}_t + \hat{H}_{t-1}, \quad (6.28)$$

$$\hat{C}_t = \lambda_Y \hat{Y}_t - \lambda_G \hat{G}_t. \quad (6.29)$$

The dependence of ambiguity on labor supply (6.24) gives rise to three key properties. First, when  $\zeta = 0$ , hours and output simply trace the movement of the exogenous government spending. In contrast, with endogenous ambiguity there is an additional AR(2) term that could potentially generate hump-shaped and persistent dynamics.

Second, endogenous uncertainty leads to co-movement in response to demand shocks. This can be analyzed by considering equation (6.25). Suppose there is a period of high labor supply triggered by an increase in government spending. Because of the negative wealth effect, the standard effect would be low consumption. However, in our model, an increase in hours raises confidence and hence agents act *as if* productivity is high. If the effect of high confidence is strong enough, the negative wealth effect could be overturned to a positive one and consumption increases as well.

Third, the model can generate countercyclical wedges. Define the labor wedge as the implicit tax that equates the marginal rate of substitution of consumption for labor with the marginal product of labor. Using the optimal condition in (6.25) we obtain

$$1 - \tau_t^H = \frac{E_{t-1}^* [C_t^{-\sigma} Z_t]}{C_t^{-\sigma} Z_t}$$

In log-linear deviations, the labor wedge is proportional to the time-varying ambiguity, which



using (6.24), makes it predictable based on past labor supply as:

$$E_{t-1}\hat{\tau}_t^H = -(\phi + \sigma\lambda_Y)\varepsilon_Z\zeta\hat{H}_{t-2}.$$

Intuitively, when there is ambiguity ( $\zeta > 0$ ) and the substitution effect is strong enough so that  $\varepsilon_Z > 0$ , labor supply at  $t - 1$  is lower as  $t - 1$  confidence is lower. From the perspective of the econometrician measuring at time  $t$  labor and consumption choices, together with measured productivity, the low labor supply is surprisingly low and can be rationalized as a high labor income tax at  $t - 1$ . In turn, the low time  $t - 1$  confidence is due to the low lagged labor supply, so the econometrician will find a systematic negative relationship between lagged hours and the labor income tax.

To understand how the model generates countercyclical wedge on assets, we analyze a decentralized version of the economy and assume that households have access to risk-free and risky assets. First, consider a risk-free bond that pays out one unit of consumption at  $t + 1$  and let  $R_t$  denote its return. As with the labor wedge, let us define an implicit tax on consumption that, using the optimality condition, becomes:

$$1 + \tau_t^B = \frac{E_t^* C_{t+1}^{-\sigma}}{E_t C_{t+1}^{-\sigma}}, \quad (6.30)$$

Here we can further explicitly show that the wedge is inversely related to labor supply:

$$\hat{\tau}_t^B = -\sigma\lambda_Y\zeta\hat{H}_{t-1}. \quad (6.31)$$

A similar logic applies to countercyclical excess return on risky assets. Consider a claim to consumption next period priced by  $Q_t^K$  :

$$Q_t^K = \beta C_t^\sigma E_t^* C_{t+1}^{1-\sigma},$$

which we can rewrite as

$$1 = \beta C_t^\sigma E_t^* [C_{t+1}^{-\sigma} R_{t+1}^K],$$

where we define the return on the claim as  $R_{t+1}^K \equiv C_{t+1}/Q_t^K$ . Under our (log-)linearized solution we get  $E_t^* R_{t+1}^K = R_t$ , where  $E_t^* R_{t+1}^K$  is the expected return on a claim to consumption under the worst-case belief. As with the consumption wedge, let us define the measured excess return wedge as  $E_t R_{t+1}^K = R_t(1 + \tau_t^K)$ , which takes the form

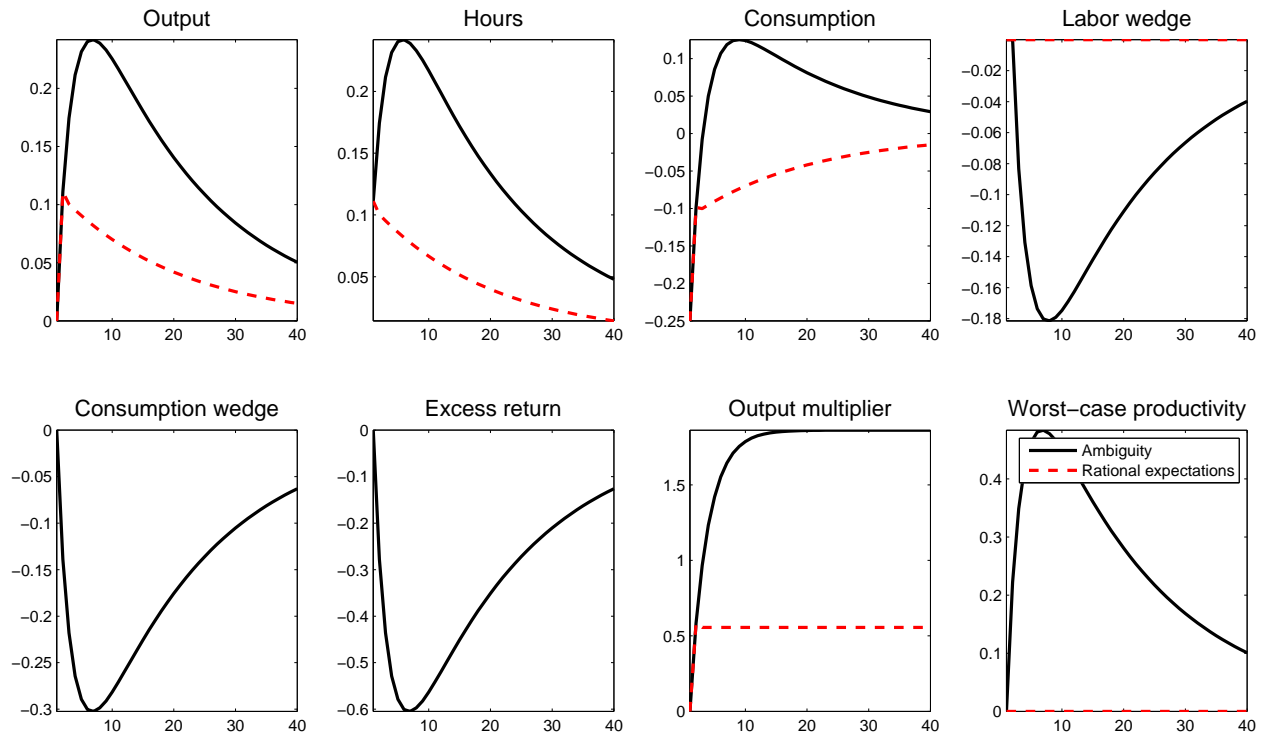
$$1 + \tau_t^K = \frac{E_t R_{t+1}^K}{E_t^* R_{t+1}^K},$$

which in turn is a function of past labor supply:

$$\hat{\tau}_t^K = -\lambda_Y \zeta \hat{H}_{t-1}. \quad (6.32)$$

Equations (6.31) and (6.32) makes transparent the predictable nature of the wedges. During periods of low confidence, driven by past low labor supply, the representative household acts *as if* future marginal utility is high. This heightened concern about future resources drives up demand for safe assets and leads to a low interest rate  $R_t$ . To rationalize the low interest rate without observing large changes in the growth rates of marginal utility, the econometrician recovers a high consumption wedge  $\tau_t^B$ . At the same time, demand for risky asset is also ‘surprisingly low’. This is rationalized by the econometrician, measuring  $R_{t+1}^K$  under the true DGP, as a high wedge  $\tau_t^K$ .

Figure 9: Stylized model: impulse response for a 1% increase in government spending



*Notes:* Output, hours, consumption, and worst-case productivity are in percent deviations from the steady state. Labor and consumption wedges and excess return are in percentage point deviations from the steady state. For the output multiplier, we plot  $dY_t/dG_t$ .

We illustrate the dynamics of this stylized model using a numerical example.<sup>48</sup> Figure 9 plots the response of endogenous variables to a 1 percent increase in government spending and compares the economy with ambiguity (black solid line) to that with rational expectations (RE, red dashed line), in which  $\zeta = 0$ . In the RE model, output and hours simply track the AR(1) evolution of exogenous government spending and consumption decreases. The labor wedge, the consumption wedge, and the ex-post excess return are zero. When ambiguity is present, output and hours show more variability and a hump-shaped response. This comes from the AR(2) dynamics for hours worked, as shown by formula (6.27). The increase in confidence (worst-case productivity) is large enough so that consumption actually increases after several periods. At the same time, the labor wedge, the consumption wedge, and the ex-post excess return are countercyclical.

The introduction of endogenous ambiguity also has an important implication regarding the size of the government spending multiplier to output. To see this consider again the case of no ambiguity ( $\zeta = 0$ ). From (6.27) and (6.28), the initial impact of a unit-increase in government spending to hours and output are given by  $\rho\varepsilon_G$  and then monotonically decreases. The government spending multiplier is given by

$$\frac{dY_t}{dG_t} \approx \frac{\lambda_Y \hat{Y}_t}{\lambda_G \hat{G}_t},$$

which, given that  $\rho\varepsilon_G < \lambda_G/\lambda_Y$ , is less than one. Indeed, in Figure 9 the multiplier stays around 0.5 in the RE model. With ambiguity, an increase in hours leads to an increase in confidence, which further raises hours over time. Because of this amplification effect, the government spending multiplier becomes well above one after a few periods. Thus, government spending has a net stimulative effect on output.

## 6.5 Quantitative model

### 6.5.1 Financial accelerator and financial shocks

We embed a Bernanke et al. (1999)-type financial accelerator mechanism by introducing an entrepreneurial sector that buys capital from households at price  $q_t$  at the end of period  $t$  and receives the proceed from production at the end of  $t+1$  and resell it to households at price  $q_{t+1}$ . Entrepreneurs are risk-neutral and hold net worth  $N_t$  which could be used to partially finance their capital expenditures  $q_t K_t$ . Entrepreneurs face an exogenous survival rate  $\zeta$ ; when they exit the market, their net worth is rebated back to the households as a lump-sum transfer. The

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<sup>48</sup>We choose parameters as follows: a ratio of government spending to output of  $g = 0.2$ ;  $\sigma = 0.5$  so the IES=2 and we pick  $\phi = 0.5$  so the Frisch elasticity of labor supply=2; a persistence of the government spending shock of  $\rho = 0.95$ ; and for the ambiguity model a feedback effect of  $\zeta = 2$ .

new entrepreneurs, who replace the entrepreneurs that exit the market, receive a start-up fund  $T_t^E$  which is financed via a lump-sum tax on households. Risk-neutral financial intermediaries provide external finance to entrepreneurs using funds obtained from households.

After the realization of period  $t + 1$  aggregate shocks, entrepreneurs sign a debt contract with the financial intermediaries. Entrepreneurs then transform capital  $K_t$  purchased from households into effective units  $\omega_{t+1}K_t$  that can be rented out to firms, where  $\omega_{t+1}$  is an idiosyncratic shock that is unobservable to the financial intermediaries unless they pay a monitoring cost. We assume that  $\omega$  is log-normally distributed with mean one:  $\ln \omega \sim N(-0.5\sigma_\omega^2, \sigma_\omega^2)$ . The loan contract is characterized by the level of capital  $q_t K_t$  and their associated level of borrowing  $B_t = q_t K_t - N_t$ , the loan rate  $Z_{t+1}$  and a cutoff value  $\bar{\omega}_{t+1}$  for the idiosyncratic shock. The indifference condition for the entrepreneurs is given by

$$\bar{\omega}_{t+1} E_{t+1}^* R_{t+1}^K q_t K_t = Z_{t+1} B_t, \quad (6.33)$$

where  $R_{t+1}^K$  is evaluated under the worst-case expectation  $E_{t+1}^*$  since the contract is signed before the resolution of firm-level uncertainty. When  $\omega_{t+1} > \bar{\omega}_{t+1}$ , entrepreneurs repay the debt to the financial intermediaries and keep the difference  $\omega_{t+1} R_{t+1}^K q_t K_t - Z_{t+1} B_t$ . When  $\omega_{t+1} \leq \bar{\omega}_{t+1}$ , entrepreneurs declare bankruptcy and repay nothing while financial intermediaries pay a monitoring cost and recover the rest  $(1 - \mu) R_{t+1}^K q_t K_t$ . The credit spread is defined as the difference between the loan rate and the risk-free rate:  $Spread_t \equiv Z_{t+1} - R_t$ .

The entrepreneur's problem is to choose  $(Z_{t+1}, B_t)$ , to maximize their payoff

$$[1 - \Gamma(\bar{\omega}_{t+1})] E_{t+1}^* R_{t+1}^K q_t K_t,$$

subject to the financial intermediaries' participation constraint (zero-profit condition), where  $\Gamma(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega + \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^\infty f(\omega) d\omega$  and  $f(\cdot)$  is the log-normal density from which  $\omega$  is drawn. The solution to the problem is characterized by the first-order condition

$$E_t^* \left\{ [1 - \Gamma(\bar{\omega}_{t+1})] \frac{R_{t+1}^K}{R_t} + \frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})} \left( \frac{R_{t+1}^K}{R_t} [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] - \Delta_t^K - 1 \right) \right\} = 0$$

, where  $G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega$  and the zero-profit condition:

$$[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] E_{t+1}^* R_{t+1}^K q_t K_t - \Delta_t^K R_t B_t = R_t B_t, \quad (6.34)$$

where  $\Delta_t^K$  is a financial shock that drives a wedge between the financial intermediaries' revenue (left-hand side) and its opportunity cost of its funds (right-hand side). Finally, the evolution

of net worth is given by

$$N_{t+1} = \zeta(1 - \Gamma(\tilde{\omega}_{t+1}))R_{t+1}^K q_t K_t + (1 - \zeta)T_t^E,$$

where  $\tilde{\omega}_{t+1}$  is the realized cutoff value, obtained by evaluating (6.33) under the realized return on capital.

### 6.5.2 Equilibrium conditions

As we describe below in Appendix 6.2, we express equilibrium conditions from the perspective of agents at both stage 1 and stage 2. At stage 1, we need not only equilibrium conditions for variable determined before production (such as utilization and hours), but also those for variables determined after production (such as consumption and investment). At stage 2, we treat variables determined before production as pre-determined. To do this, we index period  $t$  variables determined at stage 1 by  $t - 1$  and period  $t$  variables determined at stage 2 by  $t$ . We then combine stage 1 and stage 2 equilibrium conditions by using the certainty equivalence property of linearized decision rules.

We scale the variables in order to introduce stationary:

$$c_t = \frac{C_t}{\gamma^t}, y_{l,t} = \frac{Y_{l,t}}{\gamma^t}, k_{l,t-1} = \frac{K_{l,t-1}}{\gamma^t}, i_t = \frac{I_t}{\gamma^t}, w_t = \frac{W_t}{\gamma^t}, n_{t-1} = \frac{N_{t-1}}{\gamma^t}, t_t^E = \frac{T_t^E}{\gamma^t}, \tilde{\lambda}_t = \gamma^t \lambda_t, \tilde{\mu}_t = \gamma^t \mu_t,$$

where  $\mu_t$  is the Lagrangian multiplier on the capital accumulation equation. We first describe the stage 1 equilibrium conditions.

#### *Firms*

An individual firm  $l$ 's problem is to choose  $\{U_{l,t}, K_{l,t}, H_{l,t}\}$  to maximize

$$E_t^* \sum_{s=0}^{\infty} \beta^{t+s} \lambda_{t+s} [P_{t+s}^W Y_{t+s}^{\frac{1}{\theta}} Y_{l,t+s}^{1-\frac{1}{\theta}} - W_{t+s} H_{l,t+s} - r_{t+s}^K K_{l,t+s-1} - a(U_{l,t+s}) K_{l,t+s-1}],$$

where  $P_t^W$  is the price of whole-sale goods produced by firms and  $\lambda_t$ , and its detrended counterpart  $\tilde{\lambda}_t$ , is the marginal utility of the representative household:

$$\tilde{\lambda}_t = \frac{\gamma}{c_t - bc_{t-1}} - \beta b E_t^* \frac{1}{\gamma c_{t+1} - bc_t}, \quad (6.35)$$

subject to the following two constraints. The first constraint is the production function:

$$y_{l,t} = E_{t-1}^* e^{A_t + z_{l,t}} f_{l,t} \bar{\nu}_{l,t}, \quad (6.36)$$

where  $\bar{\nu}_{l,t} \equiv \sum_{j=1}^{J_{l,t}} e^{\nu_{l,j,t}} / \bar{N}$  and  $f_{l,t}$  is the input,

$$f_{l,t} = (U_{l,t} k_{l,t-1})^\alpha H_{l,t}^{1-\alpha}. \quad (6.37)$$

The worst case TFP  $E_t^* z_{l,t+1|t+1}$  is given by

$$E_t^* z_{l,t+1} = \rho_z \tilde{z}_{l,t|t} - \eta \rho_z \sqrt{\Sigma_{l,t|t}}. \quad (6.38)$$

and the Kalman filter estimate  $\tilde{z}_{l,t|t}$  evolves according to

$$\tilde{z}_{l,t|t} = \tilde{z}_{l,t|t-1} + \frac{\Sigma_{l,t|t-1}}{\Sigma_{l,t|t-1} + f_{l,t}^{-1} \sigma_\nu^2} \cdot (s_{l,t} - \tilde{z}_{l,t|t-1}). \quad (6.39)$$

The second constraint is the law of motion for posterior variance:

$$\Sigma_{l,t|t} = \left[ \frac{\sigma_\nu^2}{f_{l,t} \Sigma_{l,t|t-1} + \sigma_\nu^2} \right] \Sigma_{l,t|t-1}. \quad (6.40)$$

As described in the main text, firms take into account the impact of their input choice on worst-case probabilities.

The first-order necessary conditions for firms' input choices are as follows:

- FONC for  $\Sigma_{l,t|t}$

$$\begin{aligned} \psi_{l,t} = & \beta E_t^* \left[ \frac{1}{2} \tilde{\lambda}_{t+1} P_{t+1}^W \exp \left( A_{t+1} + \frac{\theta-1}{\theta} z_{l,t+1} \right) \left( \frac{\theta-1}{\theta} \right) \eta \rho_z \Sigma_{l,t|t}^{-\frac{1}{2}} f_{l,t+1} \right. \\ & \left. + \psi_{l,t+1} \left\{ \frac{\sigma_\nu^2 \rho_z^2}{f_{l,t+1} (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) + \sigma_\nu^2} - \frac{\sigma_\nu^2 \rho_z^2 (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) f_{l,t+1}}{\{f_{l,t+1} (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) + \sigma_\nu^2\}^2} \right\} \right], \end{aligned} \quad (6.41)$$

where  $\psi_{l,t}$  is the Lagrangian multiplier for the law of motion of posterior variance.

- FONC for  $U_{l,t}$

$$\begin{aligned} & \tilde{\lambda}_t P_t^W \left( \frac{\theta-1}{\theta} \right) \alpha \frac{y_{l,t}}{U_{l,t}} + \psi_{l,t} \frac{\alpha \sigma_\nu^2 (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2)^2 f_{l,t}}{\{f_{l,t} (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2) + \sigma_\nu^2\}^2 U_{l,t}} \\ & = \tilde{\lambda}_t \{ \chi_1 \chi_2 U_{l,t} + \chi_2 (1 - \chi_1) \} k_{l,t-1} \end{aligned} \quad (6.42)$$

- FONC for  $k_{l,t}$

$$r_t^K = P_t^W \left( \frac{\theta - 1}{\theta} \right) \alpha \frac{y_{l,t}}{k_{l,t-1}} - a(U_{l,t}) + \frac{\psi_{l,t}}{\tilde{\lambda}_t} \cdot \frac{\alpha \sigma_\nu^2 (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2)^2 f_{l,t}}{\{f_{l,t} (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2) + \sigma_\nu^2\}^2 k_{l,t-1}} \quad (6.43)$$

- FONC for  $H_{l,t}$

$$\tilde{\lambda}_t P_t^W \left( \frac{\theta - 1}{\theta} \right) (1 - \alpha) \frac{y_{l,t}}{H_{l,t}} + \psi_{l,t} \frac{(1 - \alpha) \sigma_\nu^2 (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2)^2 f_{l,t}}{\{f_{l,t} (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2) + \sigma_\nu^2\}^2 H_{l,t}} = \tilde{\lambda}_t \tilde{w}_t, \quad (6.44)$$

where  $\tilde{w}_t$  is the real wage:  $\tilde{w}_t \equiv w_t/P_t$ .

Firms sell their wholesale goods to monopolistically competitive retailers. Conditions associated with Calvo sticky prices are<sup>49</sup>

$$P_t^n = \tilde{\lambda}_t P_t^W y_t + \xi_p \beta E_t^* \left( \frac{\pi_{t+1}}{\bar{\pi}} \right)^{\theta_p} P_{t+1}^n \quad (6.45)$$

$$P_t^d = \tilde{\lambda}_t y_t + \xi_p \beta E_t^* \left( \frac{\pi_{t+1}}{\bar{\pi}} \right)^{\theta_p - 1} P_{t+1}^d \quad (6.46)$$

$$p_t^* = \left( \frac{\theta_p}{\theta_p - 1} \right) \frac{P_t^n}{P_t^d} \quad (6.47)$$

$$1 = (1 - \xi_p) (p_t^*)^{1 - \theta_p} + \xi_p \left( \frac{\bar{\pi}}{\pi_t} \right)^{1 - \theta_p} \quad (6.48)$$

$$y_t^* = \tilde{p}_t^{-\theta_p} y_t \quad (6.49)$$

$$\tilde{p}_t = (1 - \xi_p) (p_t^*)^{-\theta_p} + \xi_p \left( \frac{\bar{\pi}}{\pi_t} \right)^{-\theta_p} \quad (6.50)$$

Conditions associated with Calvo sticky wages are

$$v_t^1 = v_t^2 \quad (6.51)$$

$$v_t^1 = (w_t^*)^{1 - \theta_w} \tilde{\lambda}_t H_t \tilde{w}_t + \xi_w \beta E_t^* \left( \frac{\pi_{t+1}^w w_{t+1}^*}{\bar{\pi} w_t^*} \right)^{\theta_w - 1} v_{t+1}^1 \quad (6.52)$$

$$v_t^2 = \frac{\theta_w}{\theta_w - 1} (w_t^*)^{-\theta_w(1 + \phi)} H_t^{1 + \phi} + \xi_w \beta E_t^* \left( \frac{\pi_{t+1}^w w_{t+1}^*}{\bar{\pi} w_t^*} \right)^{\theta_w(1 + \phi)} v_{t+1}^2 \quad (6.53)$$

$$1 = (1 - \xi_w) (w_t^*)^{1 - \theta_w} + \xi_w E_t^* \left( \frac{\bar{\pi}}{\pi_t^w} \right)^{1 - \theta_w} \quad (6.54)$$

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<sup>49</sup>We eliminate  $l$ -subscripts to denote cross-sectional means (e.g.,  $y_t \equiv \int_0^1 y_{l,t} dl$ ).

$$\pi_t^w = \pi_t \tilde{w}_t / \tilde{w}_{t-1} \quad (6.55)$$

### Households

Households' Euler equation for risk-free bond:

$$\gamma \tilde{\lambda}_t = \beta E_t^* \tilde{\lambda}_{t+1} \frac{R_t}{\pi_{t+1}} \quad (6.56)$$

Households' FONC for  $i_t$

$$\begin{aligned} \gamma \tilde{\lambda}_t = & \gamma \tilde{\mu}_t \left[ 1 - \frac{\kappa}{2} \left( \frac{\gamma i_t}{i_{t-1}} - \gamma \right)^2 - \kappa \left( \frac{\gamma i_t}{i_{t-1}} - \gamma \right) \frac{\gamma i_t}{i_{t-1}} \right] \\ & + \beta E_t^* \left[ \tilde{\mu}_{t+1} \kappa \left( \frac{\gamma i_{t+1}}{i_t} - \gamma \right) \left( \frac{\gamma i_{t+1}}{i_t} \right)^2 \right] \end{aligned} \quad (6.57)$$

and the capital accumulation equation:

$$\gamma k_t = (1 - \delta) k_{t-1} + \left\{ 1 - \frac{\kappa}{2} \left( \frac{\gamma i_t}{i_{t-1}} - \gamma \right)^2 \right\} i_t. \quad (6.58)$$

### Entrepreneurial sector

Entrepreneurs' optimality condition:

$$E_t^* \left\{ [1 - \Gamma(\bar{w}_{t+1})] \frac{R_{t+1}^K}{R_t} + \frac{\Gamma'(\bar{w}_{t+1})}{\Gamma'(\bar{w}_{t+1}) - \mu G'(\bar{w}_{t+1})} \left( \frac{R_{t+1}^K}{R_t} [\Gamma(\bar{w}_{t+1}) - \mu G(\bar{w}_{t+1})] - \Delta_t^K - 1 \right) \right\} = 0 \quad (6.59)$$

and the financial intermediaries' participation constraint:

$$[\Gamma(\bar{w}_t) - \mu G(\bar{w}_t)] R_t^K q_{t-1} k_{t-1} - \Delta_{t-1}^K R_{t-1} (q_{t-1} k_{t-1} - n_{t-1}) = R_{t-1} (q_{t-1} k_{t-1} - n_{t-1}), \quad (6.60)$$

where the return on capital  $R_t^K$  is defined as

$$R_t^K = \{r_t^K + q_t(1 - \delta)\} \times \frac{\pi_t}{q_{t-1}}, \quad (6.61)$$

and

$$q_t = \tilde{\mu}_t / \tilde{\lambda}_t. \quad (6.62)$$



The law of motion of net worth is given by

$$\gamma n_t = \zeta(1 - \Gamma(\bar{\omega}_t))R_t^K q_{t-1}k_{t-1} + (1 - \zeta)t_t^E, \quad (6.63)$$

where we assume that the transfer to the new entrepreneurs is constant:  $t_t^E = t^E$ .

We use the indifference condition by the entrepreneurs to pin down the loan rate  $Z_t$ :

$$\bar{\omega}_t R_t^K q_{t-1}k_{t-1} = Z_t(q_{t-1}k_{t-1} - n_{t-1}), \quad (6.64)$$

which we use to compute the credit spread:  $Spread_t = Z_{t+1} - R_t$ .

### *Monetary policy and resource constraint*

Monetary policy rule:

$$\hat{R}_t = \sum_{i=1}^2 \rho_R^i \hat{R}_{t-i} + \sum_{i=0}^2 \phi_\pi^i \hat{\pi}_{t-i} + \sum_{i=0}^2 \phi_Y^i \Delta \hat{y}_{t-i} + \epsilon_{R,t} \quad (6.65)$$

Resource constraint:

$$c_t + i_t = (1 - \bar{g})y_t, \quad (6.66)$$

where we have ignored the small terms arising from entrepreneurial default costs.

The 32 endogenous variables we solve are:

$$k_t, y_t, i_t, c_t, H_t, U_t, f_t, \tilde{\lambda}_t, \tilde{\mu}_t, \psi_t, r_t^K, R_t, R_t^K, q_t, E_t^* z_{t+1}, \tilde{z}_{t|t}, \Sigma_{t|t}, \\ P_t^W, P_t^n, P_t^d, p_t^*, \pi_t, y_t^*, \tilde{p}_t, v_t^1, v_t^2, \tilde{w}_t, w_t^*, \pi_t^w, \bar{\omega}_t, n_t, Z_t$$

We have listed 32 conditions above, from (6.35) to (6.66). Of the above 32 endogenous variables, those that are determined at stage 1 are:

$$H_t, U_t, f_t, v_t^1, v_t^2, \tilde{w}_t, w_t^*, \pi_t^w, Z_t$$

We now describe the state 2 equilibrium conditions. To avoid repetitions, we only list conditions that are different from the state 1 conditions.

- (6.36):

$$y_{l,t} = E_{t-1}^* e^{A_t + z_{l,t}} f_{l,t-1} \bar{v}_{l,t},$$

- (6.37):

$$f_{l,t} = (U_{l,t} k_{l,t})^\alpha H_{l,t}^{1-\alpha}$$

- (6.39):

$$\tilde{z}_{l,t|t} = \tilde{z}_{l,t|t-1} + \frac{\Sigma_{l,t|t-1}}{\Sigma_{l,t|t-1} + f_{l,t-1}^{-1} \sigma_\nu^2} \cdot (s_{l,t} - \tilde{z}_{l,t|t-1})$$

- (6.40):

$$\Sigma_{l,t|t} = \left[ \frac{\sigma_\nu^2}{f_{l,t-1} \Sigma_{l,t|t-1} + \sigma_\nu^2} \right] \Sigma_{l,t|t-1}$$

- (6.41):

$$\begin{aligned} \psi_{l,t} = & \beta E_t^* \left[ \frac{1}{2} \tilde{\lambda}_{t+1} P_{t+1}^W \exp \left( A_{t+1} + \frac{\theta-1}{\theta} z_{l,t+1} \right) \left( \frac{\theta-1}{\theta} \right) \eta \rho_z \Sigma_{l,t|t}^{-\frac{1}{2}} f_{l,t} \right. \\ & \left. + \psi_{l,t+1} \left\{ \frac{\sigma_\nu^2 \rho_z^2}{f_{l,t} (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) + \sigma_\nu^2} - \frac{\sigma_\nu^2 \rho_z^2 (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) f_{l,t}}{\{f_{l,t} (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) + \sigma_\nu^2\}^2} \right\} \right] \end{aligned}$$

- (6.42):

$$\begin{aligned} & E_t^* \left[ \tilde{\lambda}_{t+1} P_{t+1}^W \left( \frac{\theta-1}{\theta} \right) \alpha \frac{y_{l,t+1}}{U_{l,t}} + \psi_{l,t+1} \frac{\alpha \sigma_\nu^2 (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2)^2 f_{l,t}}{\{f_{l,t} (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) + \sigma_\nu^2\}^2 U_{l,t}} \right] \\ & = E_t^* \tilde{\lambda}_{t+1} \{ \chi_1 \chi_2 U_{l,t} + \chi_2 (1 - \chi_1) \} k_{l,t} \end{aligned}$$

- (6.43):

$$r_t^K = P_t^W \left( \frac{\theta-1}{\theta} \right) \alpha \frac{y_{l,t}}{k_{l,t-1}} - a(U_{l,t-1}) + \frac{\psi_{l,t}}{\tilde{\lambda}_t} \cdot \frac{\alpha \sigma_\nu^2 (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2)^2 f_{l,t-1}}{\{f_{l,t-1} (\rho_z^2 \Sigma_{l,t-1|t-1} + \sigma_z^2) + \sigma_\nu^2\}^2 k_{l,t-1}}$$

- (6.44):

$$E_t^* \left[ \tilde{\lambda}_{t+1} P_{t+1}^W \left( \frac{\theta-1}{\theta} \right) (1-\alpha) \frac{y_{l,t+1}}{H_{l,t}} + \psi_{l,t+1} \frac{(1-\alpha) \sigma_\nu^2 (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) f_{l,t}}{\{f_{l,t} (\rho_z^2 \Sigma_{l,t|t} + \sigma_z^2) + \sigma_\nu^2\}^2 H_{l,t}} \right] = E_t^* \tilde{\lambda}_{t+1} \tilde{w}_t$$

- (6.52):

$$v_t^1 = (w_t^*)^{1-\theta_w} E_t^* \tilde{\lambda}_{t+1} H_t \tilde{w}_t + \xi_w \beta E_t^* \left( \frac{\pi_{t+1}^w w_{t+1}^*}{\bar{\pi} w_t^*} \right)^{\theta_w - 1} v_{t+1}^1$$

- (6.55):

$$\pi_t^w = E_t^* \pi_{t+1} \tilde{w}_t / \tilde{w}_{t-1}$$

### 6.5.3 Estimation method

We closely follow Christiano et al. (2010)'s description of the methodology. The Bayesian estimation of impulse-response matching first calculates the “likelihood” of the data using approximation based on standard asymptotic distribution theory. Let  $\hat{\psi}$  denote the impulse response function computed from an identified SVAR and let  $\psi(\theta)$  denote the impulse response function from the DSGE model, which depend on the structural parameters  $\theta$ . Suppose the DSGE model as well as the SVAR specifications are correct and let  $\theta_0$  denote the true parameter vector; hence  $\psi(\theta_0)$  is the true impulse response function. Then we have

$$\sqrt{T}(\hat{\psi} - \psi(\theta_0)) \xrightarrow{d} N(0, W(\theta_0)),$$

where  $T$  is the number of observations and  $W(\theta_0)$  is the asymptotic sampling variance, which depends on  $\theta_0$ . The asymptotic distribution of  $\hat{\psi}$  can be rewritten as

$$\hat{\psi} \xrightarrow{d} N(\psi(\theta_0), V), \quad V \equiv \frac{W(\theta_0)}{T}.$$

We use a consistent estimator of  $V$ , where the main diagonal elements consist of the sample variance of  $\hat{\psi}$ . Due to small sample considerations, the non-diagonal terms of  $V$  are set to zero.

The method then computes the likelihood

$$\mathcal{L}(\psi|\theta) = (2\pi)^{-\frac{N}{2}} |V|^{-\frac{1}{2}} \exp\{-0.5[\hat{\psi} - \psi(\theta)]' V^{-1} [\hat{\psi} - \psi(\theta)]\},$$

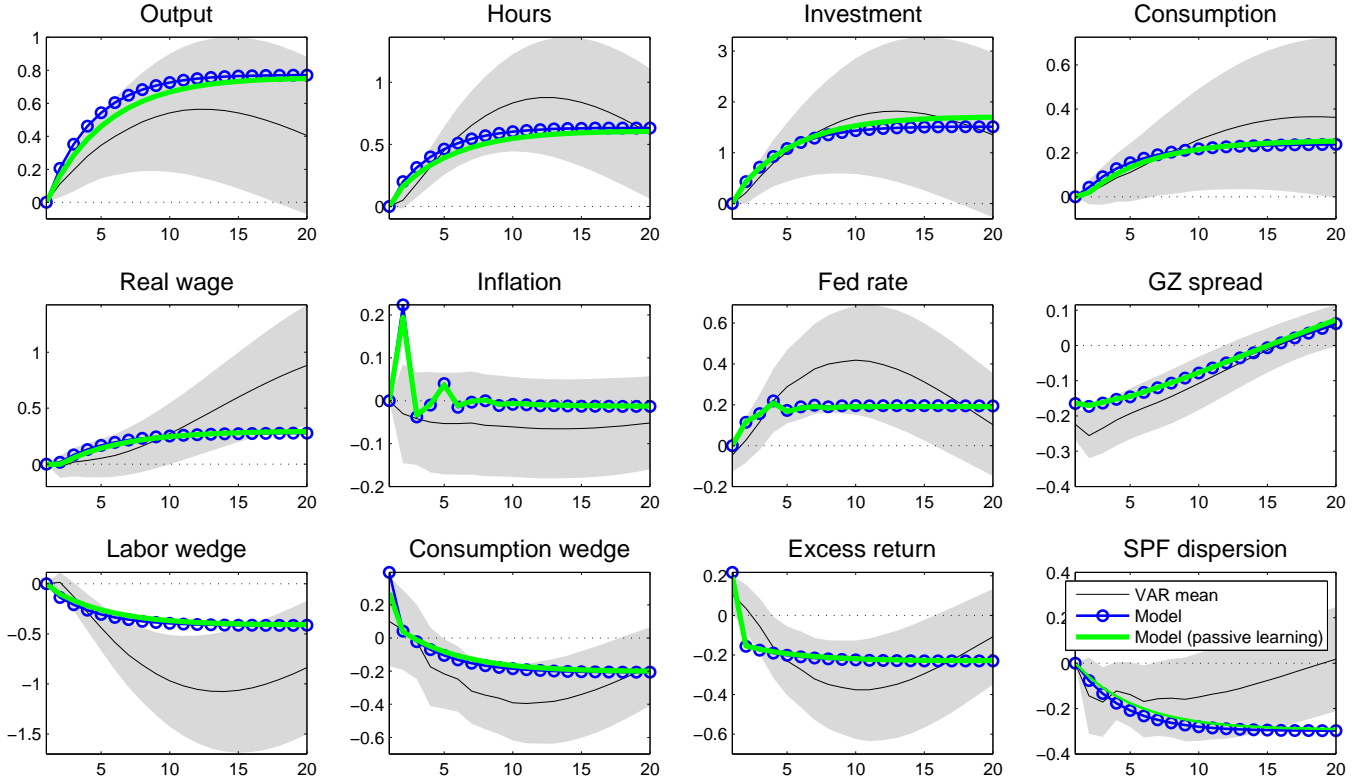
where  $N$  is the total number of elements in the impulse responses to be matched. Intuitively, the likelihood is higher when the model-based impulse response  $\psi(\theta)$  is closer to the empirical counterpart  $\hat{\psi}$ , adjusting for the precision of the estimated empirical responses. We use the Bayes law to obtain the posterior distribution  $p(\theta|\psi)$ :

$$p(\theta|\psi) = \frac{p(\theta)\mathcal{L}(\psi|\theta)}{p(\psi)},$$

where  $p(\theta)$  is the prior and  $p(\psi)$  is the marginal likelihood. We compute the posterior distribution using the random-walk Metropolis-Hastings algorithm.

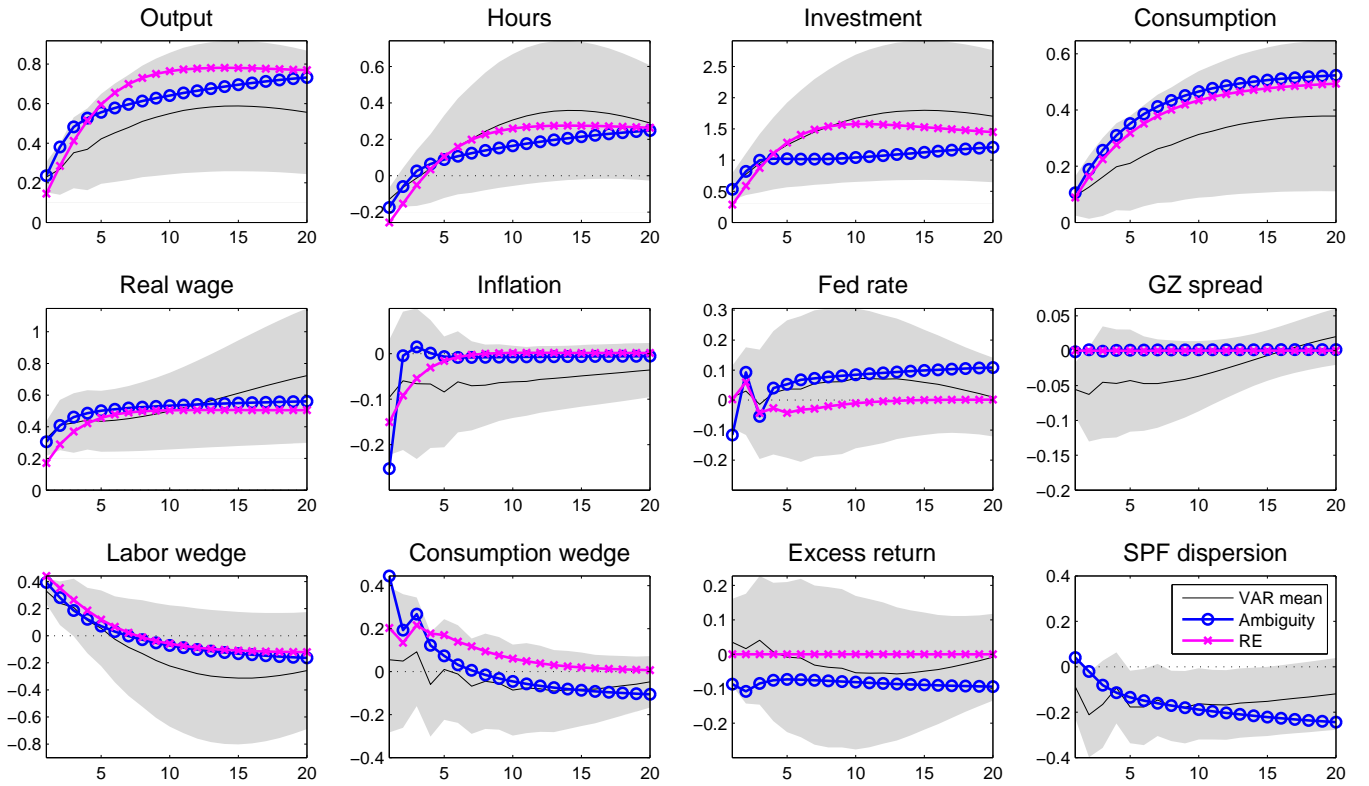
### 6.5.4 Additional figures

Figure 10: Responses to a financial shock: the role of experimentation



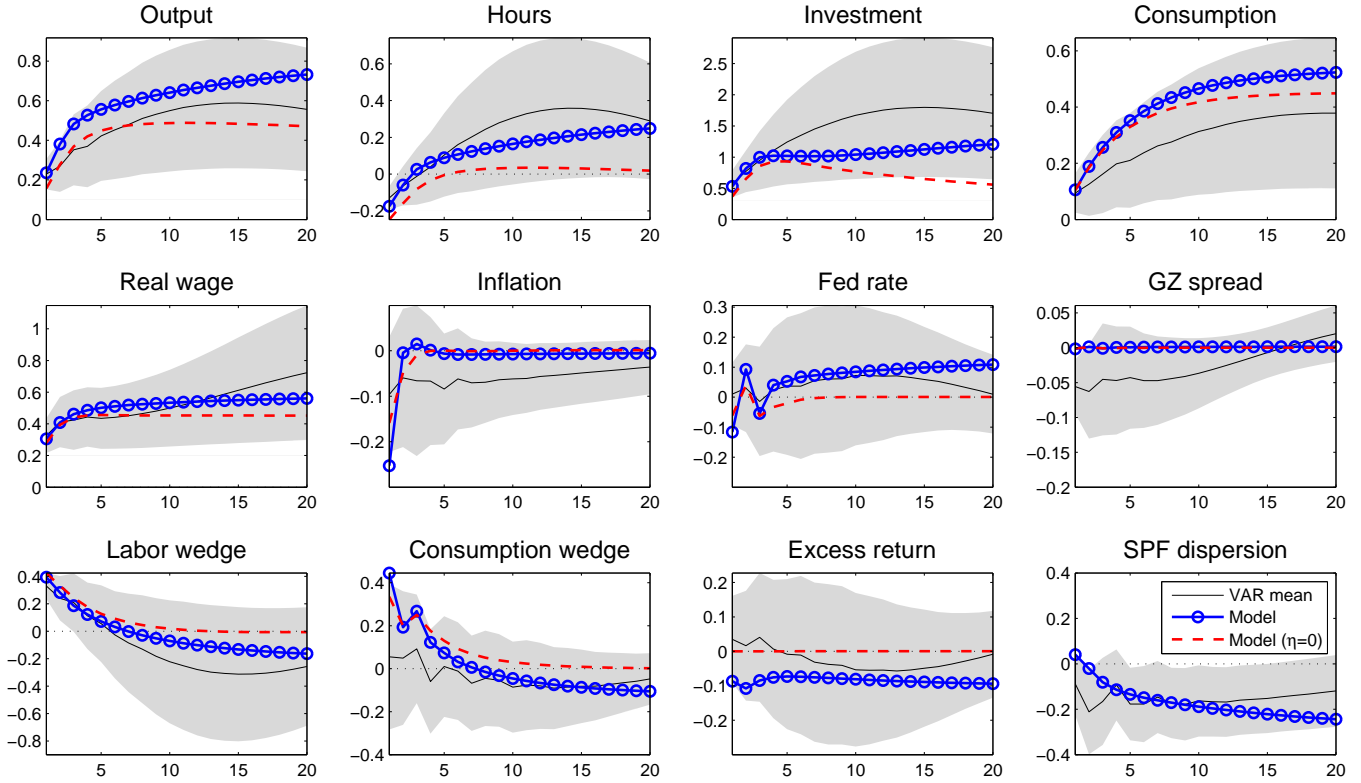
*Notes:* The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue circled lines are the impulse responses from the baseline model with ambiguity but without real and nominal rigidities. The impulse responses are estimated using only the VAR response to the financial shock. The green lines are the impulse responses from the baseline model with passive learning, where all parameter values are fixed at the estimated values in the original estimation. The responses of output, hours, investment, consumption and real wages are in percentage deviations from the steady states while inflation, Fed rate, GZ spread and excess return are in annual percentage points. The rest are in quarterly percentage points.

Figure 11: Responses to a technology shock



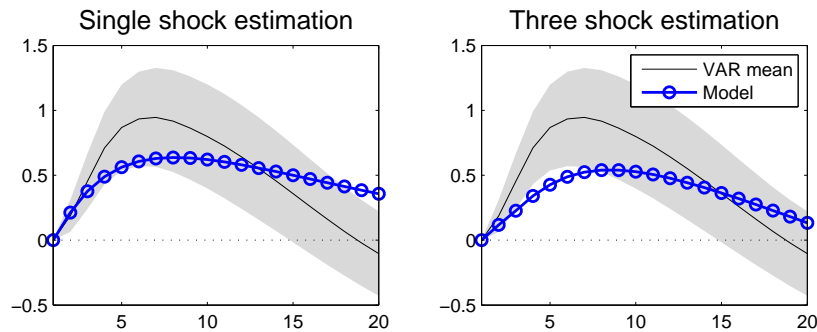
Notes: See notes from Figure 4 in the main text.

Figure 12: Responses to a technology shock: turning off ambiguity



Notes: See notes from Figure 6 in the main text.

Figure 13: The response of capital utilization to a financial shock



Notes: The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue circled lines are the impulse responses from the baseline model with ambiguity. The left panel is based on the estimation using only the VAR response to the financial shock and the right panel is based on the estimation using the responses to the VAR responses to all three structural shocks (technology, financial and monetary policy). The unit is in percentage deviations from the steady state.

## 6.6 Data sources

We use the following data:

1. Real GDP in chained dollars, BEA, NIPA table 1.1.6, line 1.
2. GDP, BEA, NIPA table 1.1.5, line 1.
3. Personal consumption expenditures on nondurables, BEA, NIPA table 1.1.5, line 5.
4. Personal consumption expenditures on services, BEA, NIPA table 1.1.5, line 6.
5. Gross private domestic fixed investment (nonresidential and residential), BEA, NIPA table 1.1.5, line 8.
6. Personal consumption expenditures on durable goods, BEA, NIPA table 1.1.5, line 4.
7. Nonfarm business hours worked, BLS PRS85006033.
8. Nonfarm business hourly compensation, BLS PRS85006103.
9. Civilian noninstitutional population (16 years and over), BLS LNU00000000.
10. Effective federal funds rate, Board of Governors of the Federal Reserve System.
11. Capacity utilization index, Board of Governors of the Federal Reserve System.
12. Credit spread (GZ spread) constructed by Gilchrist and Zakrajšek (2012).
13. Return on assets of U.S. financial corporate sector constructed by Gilchrist and Zakrajšek (2012).

We then conduct the following transformations of the above data:

14. Real per capita GDP:  $(1)/(9)$
15. GDP deflator:  $(2)/(1)$
16. Real per capita consumption:  $[(3)+(4)]/[(9)\times(15)]$
17. Real per capita investment:  $[(5)+(6)]/[(9)\times(15)]$
18. Per capita hours:  $(7)/(9)$
19. Real wages:  $(8)/(15)$